NONLINEAR VIBRATION ISOLATORS ON FINITE IMPEDANCE FOUNDATIONS

by

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CERTIFICATE

This is to certify that the present work titled NONLINEAR VIBRATION ISOLATORS ON FINITE IMPEDANCE FOUNDATIONS by Mr. Shaligram Tiwari has been carried out under my supervision and that, it has not been submitted elsewhere for a degree.

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Dedicated

to

My Parents

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I dedicate my this thesis to my parents who have given me continuous encouragement and moral support throughout my stay at IIT Kanpur.

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Shaligram Tiwari

CONTENTS

NOMENCLATURE

a_1, a_2	time independent constants
b_1, b_2	time independent constants
k	nonlinearity parameter
m	mass of the machine mounted on platelike or beamlike foundations
m_1, m_2	mass of machine and mass of foundation respectively
p	impedance ratio
q_1, q_2, q_3	coefficients of an algebraic cubic equation
r_1, r_2	coefficients of an algebraic cubic equation
t	time
x_1, x_2	absolute displacements of machine and foundation
B_o	nondimensional amplitude (time independent)
A_1, B_1	time independent constants
C	viscous damping (constant)
C_f	Coulomb or friction damping
C_{eq}	equivalent damping
$\underset{\sim}{F}$	excitation force
F_o	amplitude of exciting force
$\underset{\sim}{F}_i$	instantaneous value of force at a point
$F^o_{T_1}$	amplitude of force transmitted without the isolator
$F^o_{T_2}$	amplitude of force transmitted with the isolator
$G_1, G_2, G_3, G_4, G_5, G_6$	Coefficients of polynomial equation in nondimensional amplitude

K_1, K_2	coefficients of linear and nonlinear terms of restoring force
K_{eq}	equivalent stiffness
$\underset{\sim}{M}$	mobility of the foundation
P	power
$\underset{\sim}{V}_i$	instantaneous value of velocity at a point
X_o	Amplitude of relative displacement
Y_o	constant
Z_o	constant used in foundation impedance
$\underset{\sim}{Z}_f$	foundation impedance
RR	response ratio
α	constant (= 0.75)
β	constant (= $4/\pi$)
ϕ	phase angle
μ	mass ratio
μ^*	a function of mass ratio
ζ	viscous damping ratio
ζ_f	Coulomb damping ratio
ω_o	natural frequency (constant for every system)
ω	forcing frequency
Δ_o, Δ_1	functions of nondimensional amplitude
Ω	frequency ratio ($=\omega/\omega_o$)
Ω_n	nonresonant critical frequency for usual jump in Duffing's oscillator in absence of viscous damping

LIST OF TABLES AND FIGURES

ABSTRACT

Insertion of resilient elements or an isolator between the source and receiver of vibration is a common method of vibration control. In the present thesis, the receiver or the foundation has been modelled by (i) a rigid, finite mass, (ii) a plate and (iii) a beam. In all the cases, the internal resonances of the foundation has been neglected. The source has been represented by another rigid mass, excited by a harmonic force and undergoing rectilinear motion. The isolator is represented by a parallel combination of a nonlinear spring, a viscous damper and a Coulomb damper. The method of harmonic balance is used to obtain the harmonic response ratio assuming continuous slip at the Coulomb damper. Numerical results are presented to show the dependence of the isolator effectiveness on various parameters involved.

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The general concern with dynamical structural analysis in all its forms demonstrates the necessity for vibration control, the common objective being to minimize the vibration level. The fundamental requirement is to minimize the net vibrational power transmission from sources into the foundation and then to control the power flow or force transmitted through the structure. Typically, the system of interest consists of a machine, with or without isolators, mounted on a foundation having behaviors of pure mass like, beam like or plate like elements. The amplitude of the force transmitted from the machine to the foundation is dependent on the characteristics of the source, the isolator system and the nature of the foundation. The types of wave motion induced into the latter component are very important parameters controlling the subsequent spatial power distribution.

The simplest idealization of the vibration isolation system is a single degree-of-freedom model in which a mass is constrained to move only in one direction. The isolator is normally assumed to be massless.

However, different characteristics of the isolator and the foundation need to be considered in order to arrive at a best combination so far as the isolation is concerned. The research in vibration isolation is vast but very little attention is paid to provide guidelines to design or select the isolators on a rational basis. The general practice of isolator design is still on the old assumption that all will be well if the natural frequency of the isolator is appreciably lower than the vibration frequency to be isolated. This analysis is based on the assumptions of linear isolator characteristics and infinite foundation impedance. However, real-life isolators very often show marked non-linearity in both stiffness and damping characteristics.

Moreover, the need of considering the foundation flexibility as well as the inertia and damping offered by the foundation is apparent in many engineering situations where the machines are mounted on flexible structures such as ship decks, aircraft fuselages, car or train chasis and building floors. This thesis presents a study on nonlinear isolators on various types of finite impedance foundations.

1.2 REVIEW OF PREVIOUS WORK

The amount of literature available on the general problem of vibration isolation is too vast. In this section, the literature which has direct bearing with the present work is discussed. An overview of research work in different areas, connected with the present problem, is characterized under the following headings:

1. Modelling of finite impedance foundations
2. Analysis of a single point excitation model

1.2.1 Modelling of Finite Impedance Foundations

In the conventional analysis of vibration isolation problems, the foundation is assumed to be perfectly rigid and infinite in extent. These assumptions are, no doubt, valid in many practical problems. However, when an engine (i.e., a vibration source) is mounted on a ship deck, or an aircraft-wing or automobile chasis, the effect of the foundation has to be considered. Different types of foundations could be:

1. Soils and floors (as in buildings)
2. Beams, plates, plates stiffened with beams
3. Consisting of many structural elements (as in automobiles).

Generally the foundation is modelled by point impedances. The mobilities and damping of different types of wooden and concrete floors in occupied building in various frequency bands have been measured by White et al [1]. Snowdon [2] has considered the driving point impedance at the centre of a simply supported beam to simulate the mechanical properties of a nonrigid foundation. Later this analysis was extended to plates [3].

Machines mounted on cantilever beams and the effect of modification in the foundation structure has been studied by Wang et al [4].

Though in practice, machines are sometimes mounted on large flexible structures having so many modes of vibrations, it is not

convenient to consider each mode separately. Therefore, a simple finite seating is modelled [5] so as to find out when it can be approximated as an infinite seating for the prediction of the frequency averaged power flow.

In references [6-8], a number of typical foundations such as beams, plates etc. have been analyzed as if they were infinite. Beams and plates with force and torque excitations are studied and the resulting near and far field power flow mechanisms are examined. The driving point mobilities for these elements are expressed in terms of simple functions of frequency which yield a linear variation on log-log plots of mobility vs. frequency. A comparison between finite and infinite structures has also been made.

Goyder [9] examined different curve fitting procedures for modelling a structure. By exciting a structure at one point and measuring the frequency response at a number of positions, it is possible to construct a mathematical model of the structure. By modelling two separate components of a structure from the measured data, it is possible to obtain an estimate of the subsequent motion and power flow through the two components when coupled.

Pinnington [5] presented a model of the foundation structure which supports a motor on four isolators using point accelerances and transfer accelerances.

1.2.2 Analysis of Single Point Excitation Model

Single point excitation model is nothing but a machine (mass) moving in a single direction connected to the foundation structure

by a vibration isolator. The problem of vibration isolation of mass from an infinite impedance foundation by using a vibration isolator has been studied extensively [2]. Researchers have used different indices to express the effectiveness of a vibration isolator. Some of these are:

1. Force/displacement transmissibility (absolute)
2. Relative transmissibility
3. Response ratio
4. Isolation effectiveness
5. Insertion loss
6. Vibration power flow.
7. Vibration power flow ratio.

Log-log plots of above mentioned quantities vs. frequency ratio can be found extensively in literature for various (different) system parameters.

Snowdon [2] has studied such a model when the isolator is made of rubber-like materials for rigid as well as nonrigid foundations. The frequency dependence of dynamic modulus and damping factor of rubber-like materials is taken into consideration. The transmissibility of the system for low as well as high damping rubbers has been studied. Snowdon has used the term 'response ratio' as an index of the vibration isolation for nonrigid foundations. The nonrigid foundations considered are simply supported beams and plates and the effect of the mass loading on the response ratio has also been studied [3].

Macinante [10] considers two mass model to take care of the flexibility of floors and the transmissibility expressions (force transmitted to the structure supporting the floor) are derived in terms of nondimensional parameters of the machine, isolator and floor. The way in which the transmissibility is influenced by the two frequency ratios, viz., mounting frequency ratio and floor frequency ratio, is clearly shown as a surface in a three dimensional plot. A thorough parametric study has been conducted. The concept of vibration power flow is explained by Goyder and White [6-8]. Expressions are derived for the vibration power flow to the foundation for single point excitation, taking the foundation mobility into consideration. The constant force as well as constant velocity sources are considered. Design considerations for isolators and foundations to minimize the transmission of vibration power in the structure are examined.

Vibration isolators for machines mounted on ships are subjected to both dynamic forces involved with the working of these machines and inertial forces resulting from the rolling of the ship. In order to illustrate this better, a solution of the vibration equation of a single mass system has been analyzed, the system being excited simultaneously by a dynamic force and a motion of the foundation. It has been found that larger the ratio of rotation speed to natural frequency of vibration, larger is the relative displacement of a machine mounted on vibration isolators. Soft isolators can be used provided the frequency ratio mentioned above is chosen such that the elastic deformation of the isolators is within the allowable limits [11].

1.2.3 Analysis of Two or Multipoint Excitation Model

Analysis of a rigid body supported by undamped isolators of constant stiffness has been done for one and two planes of symmetry. Pinnington [5] has derived the expressions for vibration power flow when a motor is supported by four isolators on a flexible mounting system.

General classification of machinery and formulation of typical features of a dynamic vibro-isolation system of machines, criteria of effective isolations of main groups of machinery are given in reference [12]. Design of isolators complying with these criteria are also described.

Analysis of isolation of rigid systems supported by nonlinear (rubber-like) isolators whose stiffness and damping are frequency dependent and when the foundation has finite impedance are not presented anywhere in the literature.

In Reference [13], a pure nonlinear restoring force with viscous and Coulomb damping is considered. For an infinite foundation, the transmissibility results are derived for both force excitation and base excitation. The results are plotted as transmissibility vs. frequency ratio on log-log scale for different combinations of damping ratios (both viscous and Coulomb). The foundation is assumed to have infinite mass.

1.3 OBJECTIVES AND THE SCOPE OF THE PRESENT WORK

In the present study, the performance of nonlinear isolators on finite impedance foundations is of primary interest. The work is mainly an extension and generalization of the previous work

reported by Ravindra et al [13]. A general damping characteristic including both Coulomb and viscous types has been considered. The restoring force is taken to be a combination of a linear and cubic term.

Harmonic excitation and three different types of finite impedance foundations, viz., a pure rigid mass, platelike and beamlike, have been analysed. The foundation resonances i.e., the reflections of the transverse waves from the boundaries of the foundation have been neglected. This simplification is justified, as shown in reference [6] in the higher range of excitation frequencies.

The performance of the isolator is expressed in terms of the response ratio (in dB) as a function of the excitation frequency. The turning point frequency, beyond which multiple solutions may exist (depending on the values of various parameters), has been identified. Numerical results are obtained for different values of the parameters viz., damping, nonlinearity and foundation impedance.

CHAPTER 2

CHARACTERISTICS OF ISOLATORS AND FOUNDATIONS

2.1 PERFORMANCE INDEX OF AN ISOLATOR

Various quantities are used as pertinent indices to measure the performance of an isolator. Some of them are explained below:

1) Transmissibility (T)

The transmissibility across a system is defined as the ratio of the magnitude of the force transmitted to the foundation to that of the exciting force acting on the system. Similarly, velocity and displacement transmissibilities can be defined. Force, velocity and displacement transmissibilities are identical for a linear, single degree-of-freedom system. The transmissibility does not give the correct picture of the isolator effectiveness when the foundation is of finite impedance.

2) Response Ratio (RR)

It is defined as the ratio of the magnitude of the forces transmitted to the foundation with and without the isolator. It is identical to transmissibility, if the foundation is of infinite impedance. For a finite impedance foundation, this index provides a better description of the isolator performance.

3) Isolation Effectiveness (IE)

This is nothing but the reciprocal of the response ratio RR and a better isolator implies a higher value of IE where as T and RR decrease to signify better isolation.

4) Power Flow to Foundation (P)

$$P = \frac{1}{2} \mathrm{Re}\{\tilde{M}\} |\tilde{F}_z| = Q_f(\omega) |\tilde{F}|^2$$

where $\tilde{M}$ is the foundation mobility and $\tilde{F}_z$ is the transmitted harmonic force (both expressed in complex exponential forms), foundation, $\tilde{F}$ is the excitation force, $Q_f(w)$ is purely real and depends only on the properties of the isolator, machine and foundation. The quantity $Q_f(w)$ is used as an index of vibration isolation. It is claimed as a better representation because it includes forces as well as velocities in a single concept and takes care of the case of a finite seating area.

5) Power Flow Ratio (PF)

It is defined as the ratio of the average power transmitted to the foundation with and without the isolator. This is nothing but twice the value of the response ratio when both are expressed in dB.

All the above quantities are normally plotted on a log-log scale against the nondimensional (excitation) frequency ratio.

2.2 CHARACTERISTICS OF FOUNDATIONS

2.2.1 Foundation of Finite Impedance

Typical built-up structures such as buildings and ships and machinery foundations often consist of beams, plates and beam stiffened plates. In the present work, the characteristics of these foundations, their power transmission mechanisms are considered. The power flow formulation and the approximate structural frequency response for these foundation structures are presented in references [14, 15]. At low frequencies, discontinuities at the boundaries of these structural elements cause travelling waves to be reflected and resonances to occur. At higher frequencies, the resonant behavior is less apparent because the travelling waves are transmitted through the boundaries and power is radiated or absorbed in remote parts of the structure.

A finite structure may be approximated by an equivalent structure of infinite extent with no reflecting devices. It may be assumed that the waves propagating away from the source are attenuated by damping or radiation and are not reflected back to form standing waves. Alternatively, this is equivalent to assuming that there are many modes of vibration consisting of the motion at any one frequency without one mode being dominant.

At low frequencies, where the resonances are well separated, the above approximation is less accurate but is still valuable since it gives the average level of the response. Thus the motion of a machinery foundation may be approximated by considering it to

be of infinite extent. A number of typical foundations such as beam, plates and beam stiffened plates have, therefore, been analyzed in references [14, 15] and wave propagations in these types of structures have been studied. Force and/or torque excited foundations are considered for power flow calculations.

2.2.2 Power Flow Concepts

Power flow is the rate at which work is done and is given by

$$P_i = \underset{\sim}{F}_i \underset{\sim}{V}_i \qquad (2.2.2\text{-}1)$$

Where $\underset{\sim}{F}_i$ and $\underset{\sim}{V}_i$ are the instantaneous values of force and velocity at a point. When the power flows through an area, it is necessary to consider it as an intensity and therefore, the force $\underset{\sim}{F}_i$ is considered as the stress. With a vibrating structure, the net flow of power is more important than the instantaneous value and when both the force and velocity are harmonic, this is given by

$$P = \frac{1}{2}\frac{\omega}{2\pi}\int_0^{2\pi/\omega} \underset{\sim}{F}_i \underset{\sim}{V}_i^* \, dt \qquad (2.2.2\text{-}2)$$

where ω is the frequency of vibration. If the force and velocity are written as $\underset{\sim}{F}_i = \underset{\sim}{F}\, e^{j\omega t}$ and $\underset{\sim}{V}_i = \underset{\sim}{V}\, e^{j\omega t}$ where $\underset{\sim}{F}$ and $\underset{\sim}{V}$ are, in general, complex and may thus include a relative phase angle. Then

$$P = \frac{1}{2} |\underset{\sim}{V}| \, |\underset{\sim}{F}| \cos\theta$$

or,

$$P = \frac{1}{2}\mathrm{Re}\{\tilde{F}\tilde{V}^*\} = \frac{1}{2}\mathrm{Re}\{\tilde{F}^*\tilde{V}\} = \frac{1}{2}[\mathrm{Re}\{\tilde{F}\}\mathrm{Re}\{\tilde{V}\}$$

$$+ \mathrm{Im}\{\tilde{F}\}.\mathrm{Im}\{\tilde{V}\}] \qquad (2.2.2\text{-}3)$$

where θ is the relative phase angle and * denotes the complex conjugate. The ratio of the complex harmonic velocity to the complex harmonic force, is the mobility and this quantity is a property of the structure alone. Hence, substituting for either the force or velocity, we get

$$P = \frac{1}{2}|\tilde{F}|^2\mathrm{Re}\{\tilde{M}\} = \frac{1}{2}|\tilde{V}|\mathrm{Re}\{\tilde{M}\}/|\tilde{M}|^2 \quad \text{where } \tilde{M} = \tilde{V}/\tilde{F} \qquad (2.2.2\text{-}4)$$

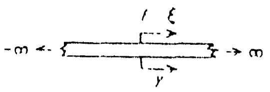
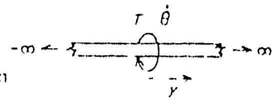
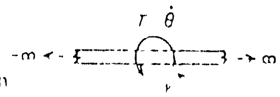
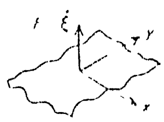
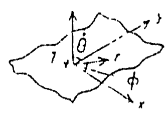
A convenient method of analysing infinite structures is to use the spatial Fourier transforms. These procedures are explained in reference [14]. The analysis is based on considering torsional, longitudinal and flexural wave motion. The driving point mobilities and expression for the input power flow with torque and force excitations are listed in Table 2.2.2-1, which has been reproduced from reference [6].

2.3 COMPARISON BETWEEN FINITE AND INFINITE FOUNDATIONS

The formulae given in Table [2.2.2-1] apply to an infinite structure in which no resonance can occur. Due to reflection from discontinuities, any finite structure will exhibit resonances which would not be apparent in the response of an infinite structure. In order to simplify the forced vibration calculations

TABLE 2.2.2-1 : (Source: Reference [6])

*Properties of infinite system (notes: *, torque applied about axis parallel to l_2, time dependence of form $e^{i\omega t}$ assumed)*

System		Driving point mobility	Power flow into system (P_s) force or torque source	Power flow into system velocity or angular velocity source
Beam longitudinal wave motion force excitation		$\frac{\dot\xi}{F} = \frac{1}{2A\sqrt{E\rho}}$	$P_s = \frac{\lvert F\rvert^2}{4A\sqrt{E\rho}}$	$P_v = \lvert\dot\xi\rvert^2 A\sqrt{E\rho}$
Beam torsional wave motion torque excitation		$\frac{\dot\theta}{T} = \frac{1}{2\sqrt{GQJ}}$	$P_s = \frac{\lvert T\rvert^2}{4\sqrt{GQJ}}$	$P_v = \lvert\dot\theta\rvert^2\sqrt{GQJ}$
Beam flexural wave motion force excitation		$\frac{\dot\xi}{F} = \frac{(1-i)}{4A\rho\sqrt{\omega}}\left(\frac{A\rho}{EI}\right)^{1/4}$	$P_s = \frac{\lvert F\rvert^2}{8A\rho\sqrt{\omega}}\left(\frac{A\rho}{EI}\right)^{1/4}$	$P_v = \lvert\dot\xi\rvert^2 A\rho\sqrt{\omega}\left(\frac{EI}{\rho A}\right)^{1/4}$
Beam flexural wave motion torque excitation		$\frac{\dot\theta}{T} = \frac{(1+i)\sqrt{\omega}}{4EI}\left(\frac{EI}{\rho A}\right)^{1/4}$	$P_s = \frac{\lvert T\rvert^2\sqrt{\omega}}{8EI}\left(\frac{EI}{\rho A}\right)^{1/4}$	$P_v = \frac{\lvert\dot\theta\rvert^2 EI}{\sqrt{\omega}}\left(\frac{\rho A}{EI}\right)^{1/4}$
Plate flexural wave motion force excitation		$\frac{\dot\xi}{F} = \frac{1}{8\sqrt{B_p\rho h}}$	$P_s = \frac{\lvert F\rvert^2}{16\sqrt{B_p\rho h}}$	$P_v = 4\lvert\dot\xi\rvert^2\sqrt{B_p\rho h}$
Plate flexural wave motion torque excitation		$\frac{\dot\theta}{T} = \frac{\omega}{8B_p(1+\nu)} \times\left[1-\frac{i4}{\pi}\ln ka + \frac{i8L}{\pi(1-\nu)}\left(\frac{h}{\pi a}\right)^2\right]$	$P_s = \frac{\omega\lvert T\rvert^2}{16B_p(1+\nu)}$	$P_v = \frac{4\lvert\dot\theta\rvert^2 B_p(1+\nu)}{\omega\left\{1+\left[\frac{4}{\pi}\ln ka - \frac{8L}{\pi(1-\nu)}\left(\frac{h}{\pi a}\right)^2\right]^2\right\}}$

System	Onset of infinite behaviour	Largest point mobility of finite system	Ratio of finite system maximum to infinite system	Wavenumber (k)	Displacement of structure
Beam longitudinal wave motion force excitation	$\omega > \frac{\pi}{\eta l}\sqrt{\left(\frac{E}{\rho}\right)}$	$\beta_l = \frac{2}{\pi A\eta\sqrt{E\rho}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{4}{\pi\eta}$	$k = \omega\sqrt{\left(\frac{\rho}{E}\right)}$	$\xi(x) = \frac{iFe^{-ikx}}{2\omega A\sqrt{E\rho}}$
Beam torsional wave motion torque excitation	$\omega > \frac{\pi}{\eta l}\sqrt{\left(\frac{GQ}{J}\right)}$	$\beta_l = \frac{2}{\pi\eta\sqrt{GQJ}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{4}{\pi\eta}$	$k = \omega\sqrt{\left(\frac{J}{GQ}\right)}$	$\theta(x) = \frac{-iTe^{-ikx}}{2\omega\sqrt{GQJ}}$
Beam flexural wave motion force excitation	$\sqrt{\omega} > \frac{4\pi}{\eta l}\left(\frac{EI}{\rho A}\right)^{1/4}$	$\beta_l = \frac{2l}{\pi^2\eta\sqrt{\rho AEI}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{4\sqrt{2}}{\pi\eta}$	$k = \sqrt{\omega}\left(\frac{\rho A}{EI}\right)^{1/4}$	$\xi(x) = \frac{iF}{4EIk^3}[e^{-ikx} - ie^{-kx}]$
Beam flexural wave motion, torque excitation	$\sqrt{\omega} > \frac{4\pi}{\eta l}\left(\frac{EI}{\rho A}\right)^{1/4}$	$\beta_l = \frac{2}{l\eta\sqrt{\rho AEI}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{2\sqrt{2}}{\pi\eta}$	$k = \sqrt{\omega}\left(\frac{\rho A}{EI}\right)^{1/4}$	$\xi(x) = \frac{T}{4EIk^2}[e^{-ikx} - e^{-kx}]$
Plate flexural wave motion force excitation	$\omega > \frac{8}{\eta l_1 l_2}\sqrt{\left(\frac{B_p}{\rho h}\right)}$	$\beta_l = \frac{4l_1l_2}{\pi^2\eta\sqrt{\rho hB_p}(l_1^2+l_2^2)}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{32l_1l_2}{\pi^2\eta(l_1^2+l_2^2)}$	$k = \sqrt{\omega}\left(\frac{\rho h}{B_p}\right)^{1/4}$	Valid in far field only $\xi(r,\phi) = \frac{-iF}{8B_pk^2}\sqrt{\left(\frac{2}{rk\pi}\right)}e^{-i(kr-\pi/4)}$
Plate flexural wave motion torque excitation	$\omega > \frac{8}{\eta l_1 l_2}\sqrt{\left(\frac{B_p}{\rho h}\right)}$	$\beta_l = \frac{16l_2}{\eta\sqrt{\rho hB_p}\,l_1(2l_2^2+l_1^2)}$		$k = \sqrt{\omega}\left(\frac{\rho h}{B_p}\right)^{1/4}$	Valid in far field only $\xi(r,\phi) = \frac{T}{8B_pk}\sqrt{\left(\frac{2}{rk\pi}\right)}e^{-i(kr+\pi/4)}\sin\phi$

in a practical case, the response estimates may be made by using the mobility and power flow formulae for infinite structures.

The largest peaks in the mobility spectra of finite beams and plates have been calculated in reference [14]. These peak mobility values represent worst cases of a finite structure being modelled as infinite. Table (2.2.2-1) contains a list of the peak mobilities and also a list of the ratios of the peak point mobility of the finite structure to the point mobility of the infinite structure written in modulus form rather than a complex quantity.

So, several types of foundations can be analyzed based on the assumption that they are infinite in extent, so long as there are no significant reflections from boundaries within the foundations. The impedance (inverse of mobility) modulus can be represented for these foundations by straight lines when plotted against frequency on log-log scale.

2.4 NATURE OF ISOLATORS & FOUNDATIONS

Isolators can be modelled as parallel combinations of

1. Linear spring - viscous damper.
2. Linear spring-combined Coulomb and viscous damping.
3. Linear-nonlinear ($K_1x+K_2x^3$) spring with combined Coulomb and viscous damping.

The first two are special cases of the third type.

Results are available in the literature for the first t cases. In the present work, the general isolator in conjunctic with various finite impedance foundations has been dealt with.

The impedance of a foundation at the point of attachmer (assuming a single point of attachment) is defined as [16]

$$\underset{\sim}{Z}_f = \frac{\underset{\sim}{F}}{\underset{\sim}{V}} \qquad (2.4-1$$

with $\underset{\sim}{F}$ as the harmonic force and $\underset{\sim}{V}$ as the harmonic velocity bot expressed in complex exponential notation.

Any foundation can be modelled to have an impedance of th form [8]

$$\underset{\sim}{Z}_f = Z_o \omega^s e^{js\pi/2} \qquad (2.4-$$

where $j = \sqrt{-1}$, ω is the frequency and Z_o is a real constant.

The modelling of different foundations is done from eqn (2.4-2) in the following ways:

(i) for a plate like foundation, $s=0 \Rightarrow \underset{\sim}{Z}_f = Z_o$

(ii) for a beam-like foundation, $s=0.5 \Rightarrow \underset{\sim}{Z}_f = Z_o \omega^{1/2} e^{j\pi/4}$

(iii) for a mass like foundation, $s=+1 \Rightarrow \underset{\sim}{Z}_f = \frac{Z_o}{\omega} e^{j\pi/2}$

(iv) for a spring-like foundation, $s=-1 \Rightarrow \underset{\sim}{Z}_f = \omega Z_o e^{-j\pi/2}$

In real-life, an isolator can be modelled with a value of s lyin in the range $-1 \leq s \leq 1$.

Expressing equation (2.4-1) in terms of real quantities, we can write

$$Z_f = \left[Z_o \, \omega^s, \quad \underline{/ \phi} \right] \tag{2.4-3}$$

where ϕ is the phase lead of the force $\underset{\sim}{F}$ from the velocity $\underset{\sim}{V}$ and is given as

$$\phi = \left(\frac{s\pi}{2} \right) \tag{2.4-4}$$

For example, if the harmonic velocity is of the form $V=V_o \cos \omega t$, then, the force is given as

$$F = Z_f \, V = Z_o V_o \omega^s \cos (\omega t + \phi) \tag{2.4-5}$$

The above expressions are valid for platelike and beamlike foundations only if $\omega < \omega_c$ because for $\omega > \omega_c$ the foundation shows an infinite behaviour [6]. Table 2.2.2-1 gives the values of critical forcing frequency beyond which the onset of infinite behaviour takes place.

CHAPTER 3

ANALYSIS OF DUFFING-TYPE ISOLATORS ON FINITE IMPEDANCE FOUNDATIONS

We shall consider in this chapter a general isolator with combined Coulomb and viscous damping and a generalized restoring force of the form $(K_1x + K_2x^3)$, where x is the deformation of the elastic element. Force excitation is considered for different foundations such as masslike, platelike or beamlike. The expressions for the response ratio and the turning point frequencies (beyond which multiple solutions exist) are derived.

3.1 MASSLIKE FOUNDATION

Figure 3.1 shows a rigid mass m_1 mounted on a platform of mass m_2. The mass m_1 is subjected to a harmonic excitation $F_o \cos \omega t$. The restoring force of the isolator is modelled as $(K_1x + K_2x^3)$, K_1 and K_2 being constants dependent upon the linear and nonlinear elastic behaviors, respectively, of the isolator. The overall damping capacity of the isolator is represented by a Coulomb and a viscous damper.

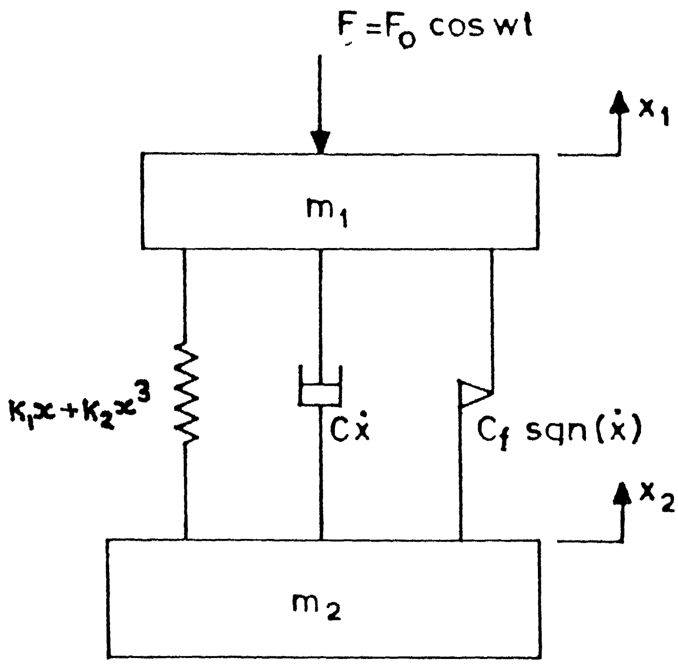


FIG. 3.1 ISOLATOR MOUNTED ON A
MASS-LIKE FOUNDATION

3.1.1 Equations of Motion

The coupled equations of motion for the system shown in Fig. 3.1 are

$$m_1\ddot{x}_1 + C(\dot{x}_1-\dot{x}_2) + C_f \,\mathrm{Sgn}\,(\dot{x}_1-\dot{x}_2) + K_1(x_1-x_2)$$
$$+ K_2\,(x_1-x_2)^3 = F_o \cos \omega t \qquad (3.1\text{-}1)$$

and

$$m_2\ddot{x}_2 - C(\dot{x}_1-\dot{x}_2) - C_f \,\mathrm{Sgn}\,(\dot{x}_1-\dot{x}_2) - K_1(x_1-x_2)$$
$$- K_2\,(x_1-x_2)^3 = 0 \qquad (3.1\text{-}2)$$

where Sgn (.) denotes the Signum function.

Equations (3.1-1) and (3.1-2) are simultaneous differential equations in x_1 and x_2 and may be solved approximately for harmonic response by putting a general solution of the form

$$x_1 = a_1 \cos \omega t + b_1 \sin \omega t$$
$$x_2 = a_2 \cos \omega t + b_2 \sin \omega t \qquad (3.1\text{-}3)$$

Equations (3.1-1) and (3.1-2) can be written as

$$\ddot{x}_1 + \frac{C}{m_1}(\dot{x}_1-\dot{x}_2) + \frac{C_f}{m_1}\,\mathrm{Sgn}\,(\dot{x}_1-\dot{x}_2) + \frac{K_1}{m_1}(x_1-x_2)$$
$$+ \frac{K_2}{m_2}(x_1-x_2)^3 = \frac{F_o}{m_1}\cos \omega t \qquad (3.1\text{-}4)$$

$$\ddot{x}_2 - \frac{C}{m_2}(\dot{x}_1-\dot{x}_2) - \frac{C_f}{m_2}\,\mathrm{Sgn}\,(\dot{x}_1-\dot{x}_2) - \frac{K_1}{m_2}(x_1-x_2)$$
$$- \frac{K_2}{m_2}(x_1-x_2)^3 = 0 \qquad (3.1\text{-}5)$$

Subtracting equation (3.1-5) from equation (3.1-4) and writing the relative displacement between the mass and the foundation (x_1-x_2) as X, we get

$$\ddot{X} + C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\dot{X} + C_f\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\mathrm{Sgn}\,(\dot{X})$$

$$+ K_1\left(\frac{1}{m_1}+\frac{1}{m_2}\right)X + K_2\left(\frac{1}{m_1}+\frac{1}{m_2}\right)X^3 = \frac{F_o}{m_1}\cos\omega t \quad (3.1\text{-}6)$$

As a first approximation, the Coulomb damping term in equation (3.1-6) can be substituted by an equivalent viscous damping [16] as explained below:

$$C_f\,\mathrm{Sgn}\,(\dot{X}) = \frac{4}{\pi}\,\frac{C_f}{\omega X_o}\,\dot{X} = \frac{\beta C_f}{\omega X_o}\,\dot{X} \quad (3.1\text{-}7)$$

where $\beta = \frac{4}{\pi}$, X_o is the amplitude of the relative motion and ω is the forcing frequency.

Also, let us introduce a term 'mass ratio' $\mu = \frac{m_2}{m_1}$ and define

$$\mu^* \equiv \frac{\mu}{1+\mu} \quad (3.1\text{-}8)$$

So, equation (3.1-6) becomes

$$\mu^* m_1 \ddot{X} + C_{eq}\dot{X} + K_1 X + K_2 X^3 = \mu^* F_o \cos\omega t \quad (3.1\text{-}9)$$

where

$$C_{eq} = C + \frac{\beta C_f}{\omega X_o} \quad (3.1\text{-}10)$$

From equation (3.1-3), $X = (a_1-a_2)\cos\omega t + (b_1-b_2)\sin\omega t$.

Let us put $a_1-a_2 \equiv A_1$ and $b_1-b_2 \equiv B_1$.

So, different terms of equation (3.1-9) have values

$$\ddot{X} = -\omega^2 (A_1 \cos \omega t + B_1 \sin \omega t)$$

$$\dot{X} = \omega B_1 \cos \omega t - \omega A_1 \sin \omega t$$

$$X^3 = \alpha A_1 (A_1{}^2+B_1{}^2) \cos \omega t + \alpha B_1 (A_1{}^2+B_1{}^2) \sin \omega t$$

$$+ \frac{1}{3} \left[\alpha A_1 (A_1{}^2+B_1{}^2) \cos 3\omega t - \alpha B_1 (A_1{}^2+B_1{}^2) \sin 3\omega t\right]$$

Substituting the above expressions in equation (3.1-9) and using the harmonic balance (with the higher harmonic terms neglected), we get

$$-\omega^2 \mu^* m_1 A_1 + \omega B_1 C_{eq} + K_1 A_1 + \alpha K_2 A_1 (A_1{}^2+B_1{}^2) = \mu^* F_o \quad (3.1\text{-}11)$$

$$-\omega^2 \mu^* m_1 B_1 - \omega A_1 C_{eq} + K_1 B_1 + \alpha K_2 B_1 (A_1{}^2+B_1{}^2) = 0 \quad (3.1\text{-}12)$$

So, obviously in equation (3.1-7),

$$X_o^2 = A_1{}^2 + B_1{}^2 \quad (3.1\text{-}13)$$

Multiplying equation (3.1-11) and equation (3.1-12) by B_1 and adding we get

$$-\mu^* m_1 \omega^2 X_o{}^2 + (K_1 + \alpha K_2 X_o{}^2)\, X_o{}^2 = F_o A_1 \mu^* \quad (3.1\text{-}14)$$

Again, multiplying equation (3.1-11) by B_1 and equation (3.1-12) by A_1 and subtracting, we get

$$\omega\, C_{eq}\, X_o{}^2 = \mu^* F_o B_1 \quad (3.1\text{-}15)$$

Squaring and adding equations (3.1-14) and (3.1-15) and substituting X_o for $(A_1{}^2 + B_1{}^2)$, we obtain

$$(\mu^* F_o)^2 = X_o{}^2 \,[(\omega\, C_{eq})^2 + \{ K_{eq} - \mu^* m_1 \omega^2 \}^2] \quad (3.1\text{-}16)$$

where

$$C_{eq} = C + \frac{\beta C_f}{\omega X_o} \; ; \quad K_{eq} = K_1 + \alpha K_2 X_o^2$$

3.1.2 Nondimensionalised Equation

Now, we can nondimensionalise equation (3.1-16) as explained below.

The following nondimensional parameters can be used:

$\Omega = \frac{\omega}{\omega_o}$, where $\omega_o = \sqrt{K_1/m_1}$ and Ω is termed as the frequency ratio.

$B_o = \frac{X_o}{Y_o}$; where $Y_o = \frac{F_o}{m\omega_o^2}$

$C = 2\zeta m_1\omega_o$; where ζ is the viscous damping ratio

$C_f = 2\zeta_f m_1\omega_o^2 Y_o$; where ζ_f is the Coulomb damping ratio

$k = \frac{F_o}{K_2 Y_o^3}$; k being the nonlinearity parameter.

Using the above parameters in equation (3.1-16), we get the following sixth degree polynomial equation in B_o (the nondimensional amplitude).

$$\alpha^2k^2B_o^6 + 2\alpha k\ (1-\mu^*\Omega^2)\ B_o^4 + (4\zeta^2\Omega^2 + \mu^{*2}\Omega^4 - 2\mu^*\Omega^2+1)\ B_o^2$$

$$+ 8\beta\zeta_f\zeta\Omega B_o + (4\beta^2\zeta_f^2 - \mu^{*2}) = 0 \qquad (3.1-17)$$

3.1.3 Critical Value of Coulomb Damping

Denoting the critical value of ζ_f at which the constant term in equation (3.1-17) goes to zero as $(\zeta_f)_c$, one can see that

$$(\zeta_f)_c = \frac{\mu^*}{2\beta} = \frac{\pi}{8}\mu^* \qquad (3.1\text{-}18)$$

With $\zeta_f = (\zeta_f)_c$, one root of amplitude equation goes to zero. As shown by Den Hertog [17] for the isolator with a linear spring and Coulomb damper, equation (3.1-17) is not valid if $\zeta_f > (\zeta_f)_c$. The approximate solution obtained by method of harmonic balance is valid only if $\zeta_f < (\zeta_f)_c$.

3.1.4 Expression for Response Ratio

Response Ratio is defined as

$$RR = \frac{\text{The amplitude of the force transmitted to the foundation with the isolator}}{\text{The amplitude of the force transmitted to the foundation without the isolator}}$$

$$= F^o_{T_2} \,/\, F^o_{T_1} \qquad (3.1\text{-}19)$$

The amplitude of the force transmitted to the foundation without the isolator

$$F^o_{T_1} = \frac{m_2}{m_1+m_2} F_o = \frac{\mu}{1+\mu} F_o = \mu^* F_o \qquad (3.1\text{-}20)$$

The amplitude of the force transmitted to the foundation with the isolator

$$F^o_{T_2} = \text{Amp.}\ [C_{eq}\ \dot{X} + K_1X + K_2X^3]$$

$$= \text{Amp.}\ [\{2m\omega_o^{\ 2}(\zeta\Omega + \frac{\beta\zeta_f}{B_o})\ (B_1\ \cos\omega t - A_1\ \sin\omega t)\}$$

$$+ \{K_1A_1 + \alpha K_2A_1(A_1^{\ 2}+B_1^{\ 2})\}\ \cos\omega t$$

$$+ \{K_1B_1 + \alpha K_2B_1(A_1^{\ 2}+B_1^{\ 2})\}\ \sin\omega t]$$

$$= \text{Amp}\ [(\omega\ C_{eq}\ B_1 + K_{eq}\ A_1)\ \cos\omega t$$

$$+ (K_{eq}\ B_1 - \omega\ C_{eq}\ A_1)\ \sin\omega t]$$

$$= (\omega^2\ C_{eq}^{\ 2} + K_{eq}^{\ 2})^{1/2}\ X_o \qquad (3.1\text{-}21)$$

Simplifying equation (3.1-21) with the help of the nondimensional parameters introduced in Sec. 3.1.2, we obtain

$$F^o_{T_2} = m\omega_o^{\ 2}X_o \left[\ 4(\zeta\Omega + \beta\zeta_f/B_o)^2 + (1+\alpha kB_o^{\ 2})^2\ \right]^{1/2} \qquad (3.1\text{-}22)$$

So, from equation (3.1-19), response ratio becomes

$$RR = \left[\ \frac{(2\zeta\Omega B_o + 2\beta\zeta_f)^2 + (B_o+\alpha kB_o^{\ 3})^2}{\mu^{*2}}\ \right]^{1/2} \qquad (3.1\text{-}23)$$

Expressing the Response Ratio in decibels (dB), we get

$$RR \equiv 10\ \log_{10} \left\{\ \frac{(2\zeta\Omega B_o + 2\beta\zeta_f)^2 + (B_o+\alpha kB_o^{\ 3})^2}{\mu^{*2}}\ \right\} \qquad (3.1\text{-}24)$$

where B_o is the root of the equation (3.1-17).

3.1.5 Multiple Solution and Turning Point

In absence of viscous damping ($\zeta = 0$), equation (3.1-17) becomes

$$\alpha^2 k^2 B_o^{\,6} + 2\alpha k\,(1-\mu^*\Omega^2)\,B_o^{\,4} + (1-\mu^*\Omega^2)^2\,B_o^{\,2} + (4\beta^2\zeta_f^{\,2} - \mu^{*2}) = 0 \tag{3.1-25}$$

If we put $B_o^{\,2} = \Delta_o$, this equation can be put in a cubic form as follows:

$$\Delta_o^{\,3} + q_1\Delta_o^{\,2} + q_2\Delta_o + q_3 = 0 \tag{3.1-26}$$

where

$$q_1 = \frac{2(1-\mu^*\Omega^2)}{\alpha k}\;;\quad q_2 = \frac{(1-\mu^*\Omega^2)^2}{\alpha^2 k^2}\;;\quad q_3 = \frac{(4\beta^2\zeta_f^{\,2} - \mu^{*2})}{\alpha^2 k^2}$$

Equation (3.1-26) can be put in standard form with the transformation $\Delta_o = (\Delta_1 - q_1/3)$, resulting in the equation

$$\Delta_1^{\,3} + r_1\Delta_1 + r_2 = 0 \tag{3.1-27}$$

where

$$r_1 = -\frac{q_1^{\,2}}{3} + q_2$$

$$r_2 = \frac{2q_1^{\,3}}{27} - \frac{q_1 q_2}{3} + q_3$$

Applying the condition of repeated roots to equation (3.1-27), we get

$$\frac{r_2^{\,2}}{4} + \frac{r_1^{\,3}}{27} = 0 \tag{3.1-28}$$

$$\Rightarrow \quad 4\beta^2\zeta_f^{\,2} - \mu^{*2} = 0 \tag{3.1-29}$$

or, $(1 - \mu^*\Omega^2)^3 = (27/4)\,\alpha k\ (4\beta^2 \zeta_f^{\,2} - \mu^{*2})$ (3.1-30)

Equation (3.1-28) gives the condition for critical Coulomb damping already derived in equation (3.1-18). The solution of equation (3.1-29) for a particular value of ζ_f, k and μ^* is the nonresonant critical frequency (Ω_n) of the usual jump observed in Duffing's equation for $\zeta = 0$. For $\Omega > \Omega_n$, we will have multiple values of response ratio for a particular value of Ω in absence of viscous damping.

3.2 PLATELIKE FOUNDATION

Figure 3.2 shows a mass m mounted on a platelike foundation with the same isolator system shown in Fig. 3.1.

3.2.1 Equations of Motion

The coupled equations of motion for Figure 3.2 are

$$m\ddot{x}_1 + C(\dot{x}_1-\dot{x}_2) + C_f\ \mathrm{Sgn}\ (\dot{x}_1-\dot{x}_2)$$

$$+ K_1(x_1-x_2) - K_2(x_1-x_2)^3 = F_o \cos \omega t \qquad (3.2\text{-}1)$$

and

$$Z_f\dot{x}_2 - C(\dot{x}_1-\dot{x}_2) - C_f\ \mathrm{Sgn}\ (\dot{x}_1-\dot{x}_2)$$

$$- K_1(x_1-x_2) - K_2(x_1-x_2)^3 = 0 \qquad (3.2\text{-}2)$$

where Sgn (.) denotes the Signum function.

Z_f is the impedance of the base at the point of attachment defined in Sec. 2.4 [16].

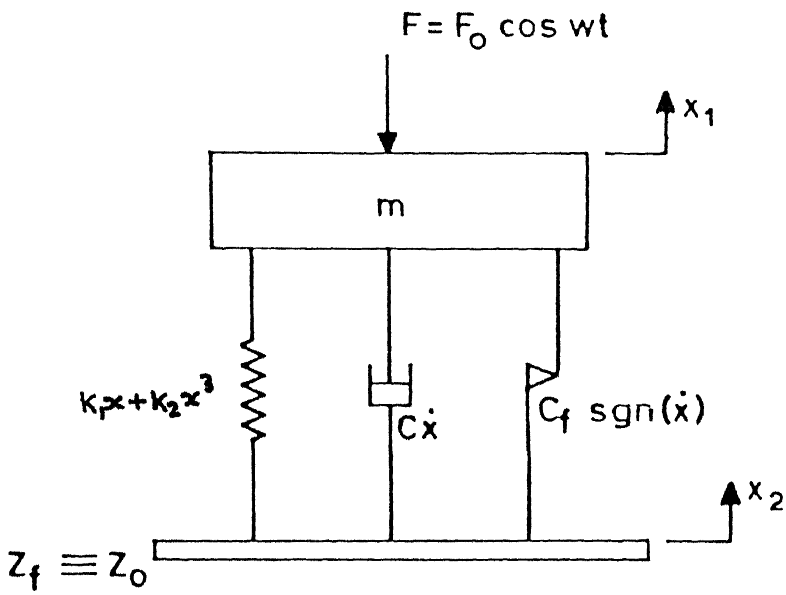


FIG.3.2 ISOLATOR MOUNTED ON A PLATE-LIKE FOUNDATION

Substituting as before

$$x_1 = a_1 \cos \omega t + b_1 \sin \omega t$$

$$x_2 = a_2 \cos \omega t + b_2 \sin \omega t$$

and carrying out the harmonic balance procedure, we obtain

$$-m\omega^2 a_1 + \omega B_1 C_{eq} + K_{eq} A_1 = F_o \qquad (3.2\text{-}3)$$

$$-m\omega^2 b_1 - \omega A_1 C_{eq} + K_{eq} B_1 = 0 \qquad (3.2\text{-}4)$$

$$Z_f \omega b_2 - \omega B_1 C_{eq} - K_{eq} A_1 = 0 \qquad (3.2\text{-}5)$$

$$-Z_f \omega a_2 + \omega A_1 C_{eq} - K_{eq} B_1 = 0 \qquad (3.2\text{-}6)$$

where, $A_1 \equiv a_1 - a_2$; $B_1 \equiv b_1 - b_2$.

Multiplying equation (3.2-3) by A_1 and equation (3.2-4) by B_1 and adding, we get

$$a_1 A_1 + b_1 B_1 = \frac{X_o^2}{m\omega^2} K_{eq} - \frac{F_o}{m\omega^2} A_1 \qquad (3.2\text{-}7)$$

Multiplying equation (3.2-5) by B_1 and equation (3.2-6) by A_1 and subtracting, we get

$$b_1 A_1 + a_2 B_1 = \frac{C_{eq}}{Z_f} X_o^2 \qquad (3.2\text{-}8)$$

Subtracting equation (3.2-8) from equation (3.2-7), we get

$$X_o^2 = X_o^2 \left[\frac{K_{eq}}{m\omega^2} - \frac{C_{eq}}{Z_f} \right] - \frac{F_o}{m\omega^2} A_1$$

$$\frac{F_o}{m\omega^2} A_1 = X_o^2 \left[\frac{K_{eq}}{m\omega^2} - \frac{C_{eq}}{Z_f} - 1 \right] \qquad (3.2\text{-}9)$$

Again, multiplying equation (3.2-4) by B_1 and equation (3.2-5) by A_1 and subtracting latter from former, we get

$$a_1B_1 - b_1A_1 = \frac{1}{m\omega^2}\left[\omega X_o^2 C_{eq} - F_oB_1\right] \qquad (3.2\text{-}10)$$

Multiplying equation (3.2-6) by A_1 and equation (3.2-7) by B_1 and adding, we get

$$b_2 A_1 - a_2B_1 = \frac{K_{eq}}{\omega Z_f} X_o^2 \qquad (3.2\text{-}11)$$

Adding equations (3.2-10) and (3.2-11), we get

$$\frac{F_o}{m\omega^2} B_1 = X_o^2 \left[\frac{\omega C_{eq}}{m\omega^2} + \frac{K_{eq}}{\omega Z_f} \right] \qquad (3.2\text{-}12)$$

Squaring and adding equations (3.2-9) and (3.2-12), we obtain

$$\left(\frac{F_o}{m\omega^2} \right)^2 = X_o^2 \left[\left(\frac{K_{eq}}{m\omega^2} - \frac{C_{eq}}{Z_f} - 1\right)^2 + \left(\frac{\omega C_{eq}}{m\omega^2} + \frac{K_{eq}}{\omega Z_f} \right)^2\right]$$

(3.2-13)

As discussed in Sec. 2.4, the nature of Z_f for platelike foundations is of the form [8]

$$Z_f = Z_o \text{ and } \phi = 0 \qquad (3.2\text{-}14)$$

3.2.2 Nondimensionalised Equation

Equation (3.2-13) can be nondimensionalised as explained below.

As already done in Sec. (3.1.2), let us introduce the nondimensional parameters:

$\Omega = \frac{\omega}{\omega_o}$, where $\omega_o \sqrt{K_1/m}$

$B_o = X_o/Y_o$;

where X_o is the amplitude of relative displacement

$C = 2\zeta m\omega_o$; ζ being the viscous damping ratio

$C_f = 2\zeta_f\, m\omega_o^2 Y_o$; ζ_f being the Coulomb damping ratio

$$k = \frac{F_o}{K_2\, Y_o^3} = \frac{m\,\omega_o^2}{K_2\, Y_o^2}$$

$Z_f = Z_o = 2\, pm\omega_o$; p being defined as the impedance ratio.

Putting in equation (3.2-13), we get the sixth degree polynomial equation in B_o (the nondimensional amplitude), as

$$\begin{aligned} &\alpha^2 k^2\, (4p^2+\Omega^2)\, B_o^6 + 2\alpha k\, [\, 4p(p-\zeta\Omega^2-p\Omega^2) + \Omega\, (4p\zeta\Omega+\Omega)]\, B_o^4 \\ &+ [4(p-\zeta\Omega^2-p\Omega^2)^2 + (4p\zeta\Omega+\Omega)^2]\, B_o^2 \\ &+ 8\, \zeta_f\beta\, [p(4p\zeta\Omega+\Omega) - \Omega\, (p-\zeta\Omega^2-p\Omega^2)]\, B_o \\ &+ 4\, [\zeta_f^2\beta^2\, (\Omega^2+4p^2) - p^2] = 0 \end{aligned} \qquad (3.2\text{-}15)$$

3.2.3 Critical Value of Coulomb Damping

Equating constant term of equation (3.2-15) equal to zero, we get as in Sec. 3.1.3,

$$(\zeta_f)_c = \frac{\pi p}{4\sqrt{\Omega^2 + 4p^2}} \qquad (3.2\text{-}16)$$

Unlike masslike foundation case, here $(\zeta_f)_c$ is frequency dependent. As mentioned before, the approximate analysis of harmonic balance is not valid for $\zeta > (\zeta_f)_c$.

3.2.4 Expression for Response Ratio

The formula for the response ratio (RR) is same as in Sec. 3.1.3.

$$RR = F^o_{T_2} \,/\, F^o_{T_1} \qquad (3.2\text{-}17)$$

The amplitude of the force transmitted to the foundation without the isolator

$$F^o_{T_1} = \left| \frac{Z_o}{Z_{eq}} \right| F_o = \frac{Z_o F_o}{\sqrt{Z_o^2 + m\omega^2}} \qquad (3.2\text{-}18)$$

where $|\cdot|$ gives the magnitude of complex impedance. (Sec. 2.4).

With the help of the nondimensional parameters defined in Sec. (3.2.2), equation (3.2-18) reduces to the form

$$F^o_{T_1} = \frac{2p}{\sqrt{4p^2+\Omega^2}} F_o \qquad (3.2\text{-}19)$$

Again, the amplitude of the force transmitted to the foundation with the isolator $F^o_{T_2}$ remains identical with equation (3.1-21) which gives $F^o_{T_2}$ for masslike foundation.

Hence, $$F^o_{T_2} = \sqrt{\omega^2 C_{eq}{}^2 + K_{eq}{}^2}\; X_o \qquad (3.2\text{-}20)$$

Nondimensionalizing the expression on R.H.S. of equation (3.2-20), we obtain

$$F^o_{T_2} = F_o B_o \left[4(\zeta\Omega + \beta\zeta_f/B_o)^2 + (1+\alpha k B_o{}^2)^2 \right]^{1/2} \qquad (3.2\text{-}21)$$

Now, equation (3.2-17) gives the expression for the response ratio (RR) as

$$RR = \left[\frac{(4p^2+\Omega^2)\left\{ (1+\alpha k B_o{}^2)^2 + 4(\zeta\Omega + \frac{\beta\zeta_f}{B_o})^2 \right\} B_o{}^2}{4p^2} \right]^{1/2} \qquad (3.2\text{-}22)$$

Expressing the response ratio in dB, we get

$$RR = 10 \log_{10} \left[\frac{(4p^2+\Omega^2)\left\{ (1+\alpha k B_o{}^2)^2 + 4(\zeta\Omega + \frac{\beta\zeta_f}{B_o})^2 \right\} B_o{}^2}{4p^2} \right]$$

(3.2-23)

where B_o is the root of the equation (3.2-15).

3.2.5 Multiple Solution and Turning Point

In section (3.1.5) we derived the condition for multiple roots of equation (3.1-17) for a masslike foundation in absence of viscous damping ($\zeta = 0$) by considering it to be a cubic in $B_o{}^2$. But in equation (3.2-15) by putting $\zeta = 0$, we are still left with the coefficient of B_o as nonzero. So, it is not so easy to derive analytically the expression for nonresonant critical frequency (Ω_n) for platelike foundations.

3.3 BEAM-LIKE FOUNDATION

Figure 3.3 shows a mass m mounted on a beam-like foundation with the same isolators shown in Fig. 3.1 and Fig. 3.2.

As explained in Sec. 2.4, the expression for Z_f for beamlike foundation is obtained from the expression

$$Z_f = Z_o\, \omega^s \; \underline{/\phi}$$

For beamlike foundation, s = 0.5; and so

$$Z_f = Z_o\, \omega^{1/2} \; \underline{/\ \pi/4} \qquad (3.3\text{-}1)$$

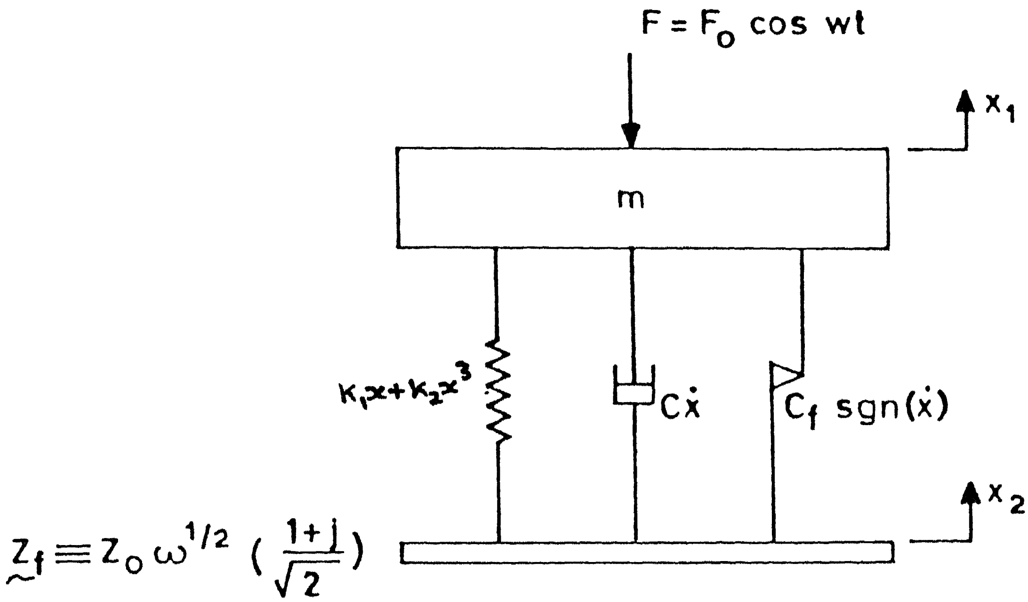


FIG.3.3 ISOLATOR MOUNTED ON A BEAM-LIKE FOUNDATION

3.3.1 Equations of Motion

The coupled equations of motion for the system shown in Fig. 3.3 are same as equations (3.2-1) and (3.2-2).

$$m\ddot{x}_1 + C(\dot{x}_1-\dot{x}_2) + C_f \text{ Sgn } (\dot{x}_1-\dot{x}_2)$$

$$+ K_1(x_1-x_2) - K_2(x_1-x_2)^3 = F_o \cos \omega t \qquad (3.2\text{-}1)$$

and

$$Z_f\dot{x}_2 - C(\dot{x}_1-\dot{x}_2) - C_f \text{ Sgn } (\dot{x}_1-\dot{x}_2)$$

$$- K_1(x_1-x_2) - K_2(x_1-x_2)^3 = 0 \qquad (3.2\text{-}2)$$

where Sgn (.) denotes the Signum function.

Substituting as before

$$x_1 = a_1 \cos \omega t + b_1 \sin \omega t$$

$$x_2 = a_2 \cos \omega t + b_2 \sin \omega t$$

and carrying out the harmonic balance procedure, with the phase lead of Z_f being considered, we obtain

$$-m\omega^2 a_1 + \omega B_1 C_{eq} + K_{eq} A_1 = F_o \qquad (3.3\text{-}4)$$

$$-m\omega^2 b_1 - \omega A_1 C_{eq} + K_{eq} B_1 = 0 \qquad (3.3\text{-}5)$$

$$\frac{Z_o\omega^{3/2}}{\sqrt{2}} (a_2-b_2) + \omega B_1 C_{eq} + K_{eq} A_1 = 0 \qquad (3.3\text{-}6)$$

$$\frac{Z_o\omega^{3/2}}{\sqrt{2}} (a_2-b_2) + \omega A_1 C_{eq} + K_{eq} B_1 = 0 \qquad (3.3\text{-}7)$$

Carrying out a similar algebra as explained in Secs.,3.1 and 3.2 for the above system of equations, we finally get the following reduced equation

$$2\left(\frac{F_o}{m\omega^2}\right)^2 = X_o^2\left[\left(\frac{K_{eq}}{m\omega^2} + \frac{\sqrt{2}\,K_{eq}}{Z_o\,\omega^{3/2}} + \frac{C_{eq}}{m\omega} - 1\right)^2 + \left(\frac{K_{eq}}{m\omega^2} - \frac{C_{eq}}{m\omega} - \frac{\sqrt{2}\,C_{eq}}{Z_o\,\omega^{1/2}} - 1\right)^2\right] \qquad (3.3\text{-}8)$$

3.3.2 Nondimensionalised Equation

Equation (3.2-8) can be nondimensionalised as explained below:

As usual, like masslike and platelike foundations, the following nondimensional parameters are introduced:

$\Omega = \frac{\omega}{\omega_o}$; $\omega_o = \sqrt{K_1/m_o}$

$B_o = X_o/Y_o$;

where X_o is the amplitude of relative displacement

and $Y_o = F_o/m\omega_o^2$

$C = 2\zeta\, m\omega_o$

$C_f = 2\zeta_f\, m\omega_o^2\, Y_o$

$k = \frac{F}{K_2\, Y_o^3} = \frac{m\omega_o^2}{K_2\, Y_o^2}$

$Z_o = 2pm\,\omega_o^{1/2}$; p being again defined as the impedance ratio.

The impedance ratio here is different from that introduced for a platelike foundation because of obvious difference in nondimensionalizing.

Putting in equation (3.3-8) and carrying out the required algebra, we finally obtain the sixth degree polynomial equation B_o

(the nondimensional amplitude), as

$$G_1 B_o^6 + G_2 B_o^4 + G_3 B_o^3 + G_4 B_o^2 + G_5 B_o + G_6 = 0 \qquad (3.3\text{-}9)$$

where,

$$G_1 = \alpha^2 k^2 \{4p^2 + (2p+\sqrt{2}\,\Omega^{1/2})^2\}$$

$$G_2 = 2\alpha k \left\{4p\left[p-4p\zeta\Omega - \sqrt{2}\,\zeta\Omega^{3/2} - p\Omega^2\right] + \left(2p+\sqrt{2}\,\Omega^{1/2}\right)\left(2p+\sqrt{2}\,\Omega^{1/2} + 4\zeta\Omega - 2p\Omega^2\right)\right\}$$

$$G_3 = 8\alpha k \left\{\beta\zeta_f \left[2p + \sqrt{2}\,\Omega^{1/2}\right] - 4p\zeta_f\beta \left(4p + \sqrt{2}\,\Omega^{1/2}\right)\right\}$$

$$G_4 = \left\{\left[2p + \sqrt{2}\,\Omega^{1/2} + 4\zeta\Omega - 2p\Omega^2\right]^2 + 4\left(p - 4p\,\zeta\Omega^{3/2} - p\Omega^2\right)^2\right\}$$

$$G_5 = 8\beta\,\zeta_f \left\{\left[2p + \sqrt{2}\,\Omega^{1/2} + 4\zeta\Omega - 2p\Omega^2\right] - \left[4p+\sqrt{2}\,\Omega^{1/2}\right] \times \left[p - 4p\zeta\Omega - \sqrt{2}\,\zeta\Omega^{3/2} - p\Omega^2\right]\right\}$$

$$G_6 = 4\left\{\beta^2 \zeta_f^2 \left[4 + \left[4p + \sqrt{2}\,\Omega^{1/2}\right]^2\right] - 2p^2\right\}$$

3.3.3 Critical Value of Coulomb Damping

As discussed in Sec. 3.2.3, the critical value of Coulomb damping from equation (3.3-9) is obtained by equating constant term to zero.

$$(\zeta_f)_c = \frac{\pi\sqrt{2}\,p}{4\left(4 + \left(4p + \sqrt{2}\,\Omega^{1/2}\right)^2\right)^{1/2}} \qquad (3.3\text{-}10)$$

The approximate analysis of harmonic balance is not valid for $\zeta > (\zeta_f)_c$.

3.3.4 Expression for Response Ratio

The formula for the response ratio (RR) is same as in Sec. (3.1.3) and (3.2.3)

$$RR = F^{o}_{T_2} \;/\; F^{o}_{T_1} \tag{3.3-11}$$

The amplitude of the force transmitted to the foundation without the isolator

$$F^{o}_{T_1} = \left(\frac{\sqrt{2}\,p \cdot \sqrt{2}}{2p^2 + (\sqrt{2}p + \Omega^{1/2})^2} \right) F_o .$$

$$F^{o}_{T_1} = \left(\frac{2p}{(4p^2 + 2\sqrt{2}\,p\,\Omega^{1/2} + \Omega)^{1/2}} \right) F_o \tag{3.3-12}$$

The amplitude of force transmitted to the foundation with the isolator is identical in form to equation (3.1-21) and (3.2-23) and becomes

$$F^{o}_{T_2} = B_o \left[\left(1 + \alpha k B_o^2 \right)^2 + 4 \left(\zeta\Omega + \frac{\beta\zeta_f}{B_o} \right)^2 \right]^{1/2} F_o \tag{3.3-13}$$

So, from equation (3.3-10), the expression for the response ratio becomes

$$RR = \left[\frac{B_o^2 \left(4p^2 + 2\sqrt{2}p\Omega^{1/2}\right) \left\{ \left(1 + \alpha k B_o^2\right)^2 + 4 \left(\zeta\Omega + \frac{\beta\zeta_f}{B_o}\right)^2 \right\}}{4p^2} \right]^{1/2} \tag{3.3-14}$$

In decibels (dB), the expression for response ratio can be written as

$$RR = 10 \log_{10} \left[\frac{B_o^2 \left(4p^2 + 2\sqrt{2}p\Omega^{1/2}\right) \left\{ \left(1 + \alpha k B_o^2\right)^2 + 4 \left(\zeta\Omega + \frac{\beta\zeta_f}{B_o}\right)^2 \right\}}{4p^2} \right. \tag{3.3-15}$$

3.3.5 Multiple Solution and Turning Point

As pointed out in Section (3.2.5) for platelike foundation, we can see that in case of beamlike foundation also, in the absence of viscous damping (ζ = 0) in equation (3.3-9) the coefficients of B_o^3 and B_o are nonzero. Hence, analytically it will be quite tedious to derive the condition for repeated roots of equation (3.3-9) with $\zeta = 0$. In other words, even in beamlike foundations the analytical evaluation of Ω_n is not easy.

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 INTRODUCTION

In this chapter, the expressions derived in Chapter 3 for the response ratio have been computed numerically and plots of response ratio vs. frequency ratio are presented. The sixth degree polynomial equations in B_0(the nondimensional amplitude) are solved numerically and the positive real values of B_0 are used to compute the response ratio expressions for masslike, platelike and beamlike foundations respectively. The NAG routine available in Computer Centre of IIT Kanpur was used for solving the polynomial equations.

4.2 MASSLIKE FOUNDATION

The polynomial equation in B_0 of equation (3.1-16) was solved numerically with particular set of values of parameters ζ, ζ_f, k and μ^* for different values of Ω and the computed response ratio in dB (20 $\log_{10}$ RR) versus frequency ratio (Ω) plots for various set of values of parameters are presented. These plots show the performance characteristics of the isolator.

4.2.1 Foundation with Infinite Mass

If in eqn. (3.1-8), foundation mass $m_2 \rightarrow \infty$, we get $\mu^* \rightarrow 1$. Reference [13] gives performance characteristics of such isolators and the results are presented through the plots of transmissibility vs. frequency ratio for force excitation as well as base excitation. But, this work considers a pure nonlinear restoring force of the form KX^3 and the effect of the linear part of the restoring force is not considered.

In the present work, a general restoring force of the form $(K_1x + K_2x^3)$ is considered and the nonlinearity is expressed by the nonlinearity parameter k. Figures 4.1–4.4 show the response ratio vs. frequency ratio plots for various values of ζ, ζ_f, and k for the case $\mu^* = 1$.

It can be seen that the usual jump present in a hard, Duffing equation is present, specially in the absence of viscous damping. There are two stable branches and one unstable branch (unstable branch shown dotted) in the curve and in the absence of viscous damping, the resonant branch extends to infinity (Fig. 4.1 and 4.2). However, in presence of viscous damping, the resonant response ratio can be controlled as shown in Figures 4.3 and 4.4. It can also be noted that the response ratio at high frequencies is almost constant in the absence of viscous damping but in presence of viscous damping it does not remain constant. Figures 4.3 and 4.4 show that with increase in both ζ and ζ_f simultaneously, the high frequency response ratio increases. In

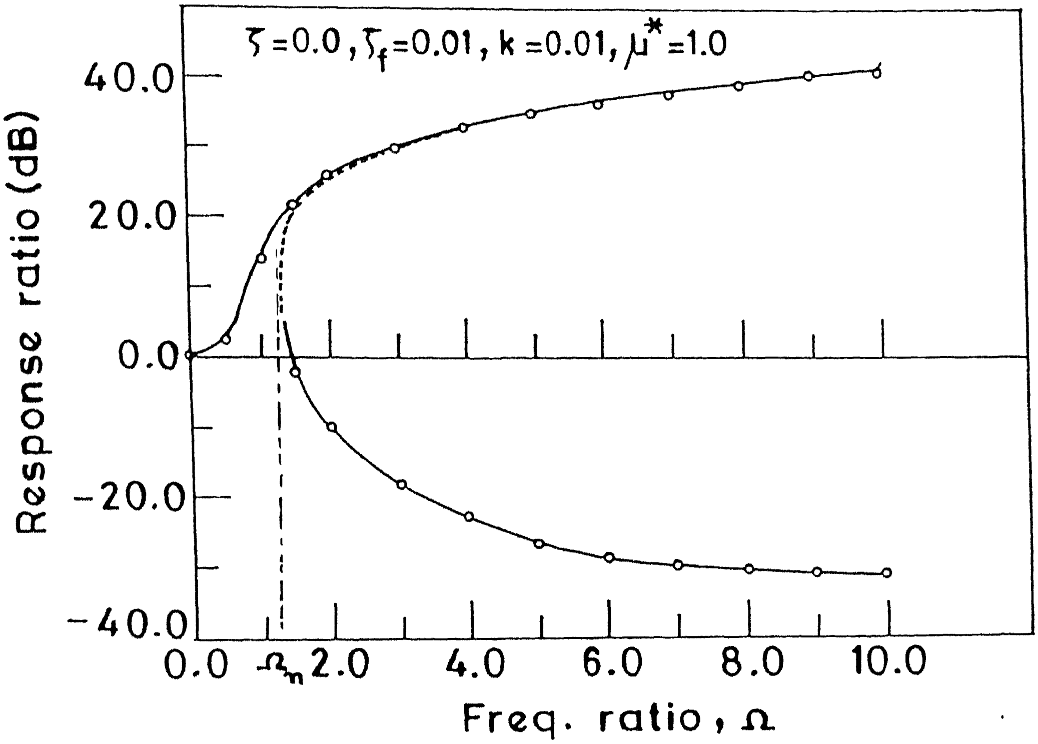


Fig.4.1(a) Response of mass-like foundation

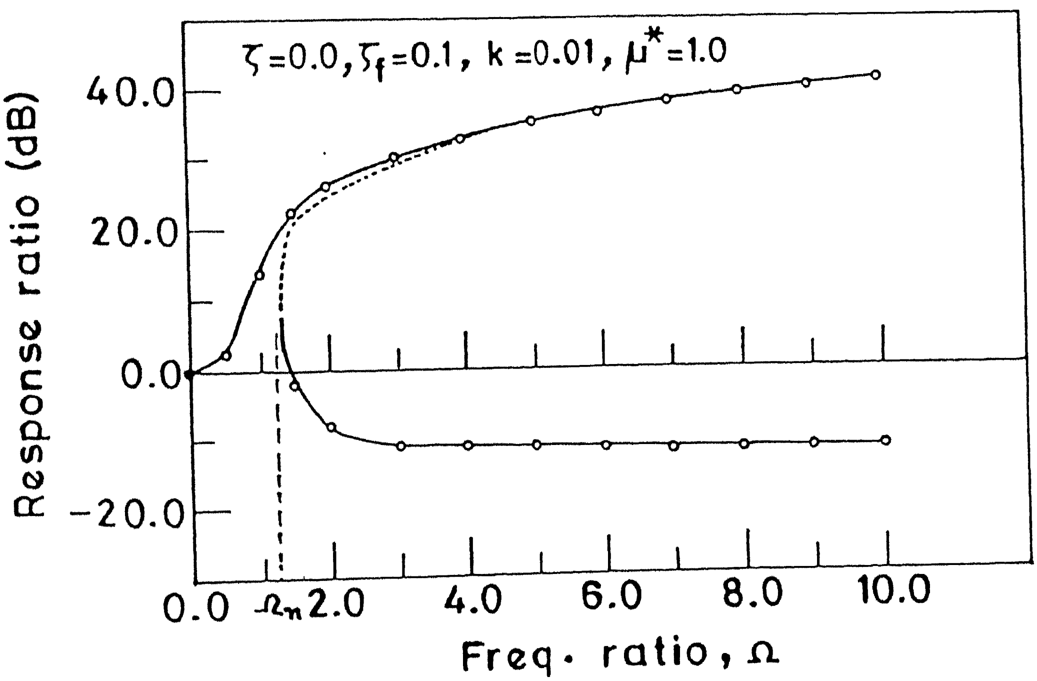


Fig.4.1(b) Response of mass-like foundation

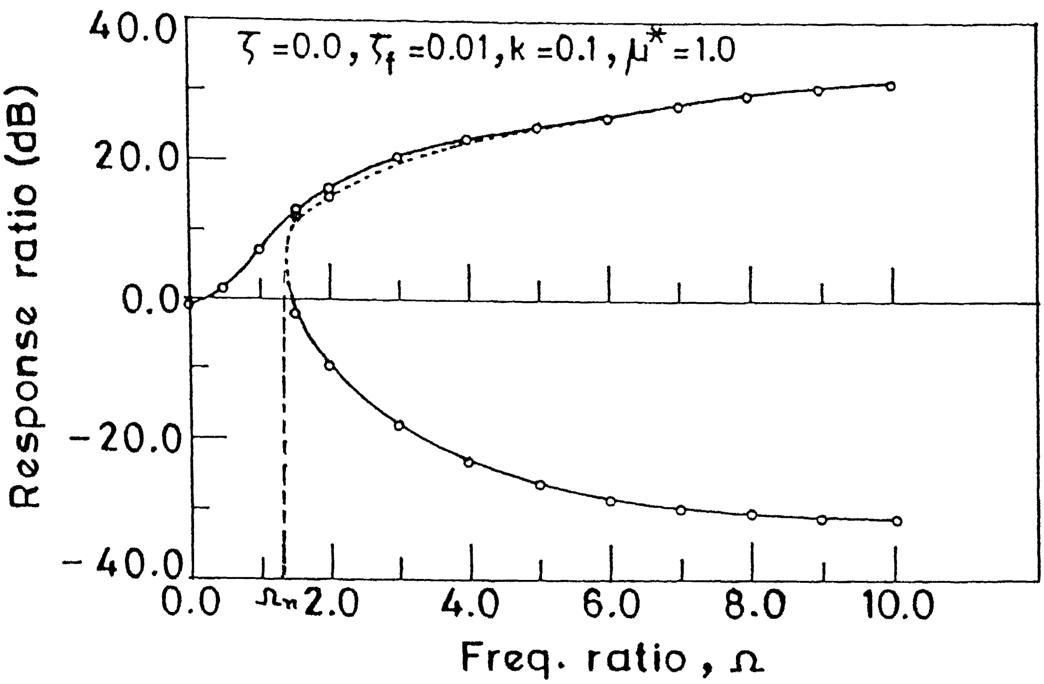


Fig.4.2(a) Response of mass-like foundation

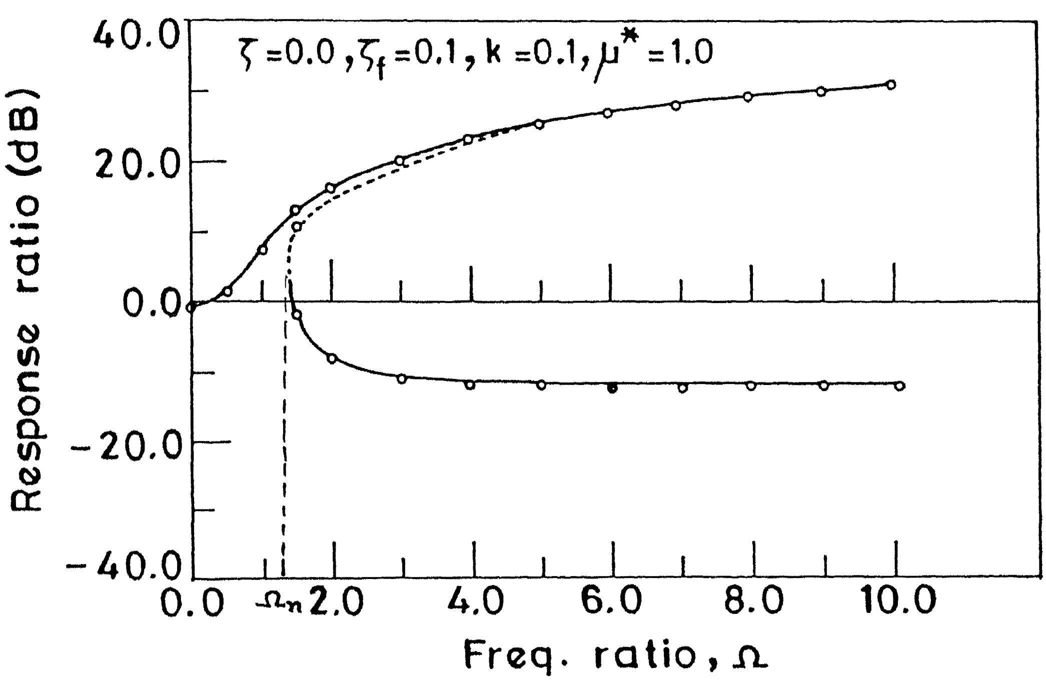


Fig.4.2(b) Response of mass-like foundation

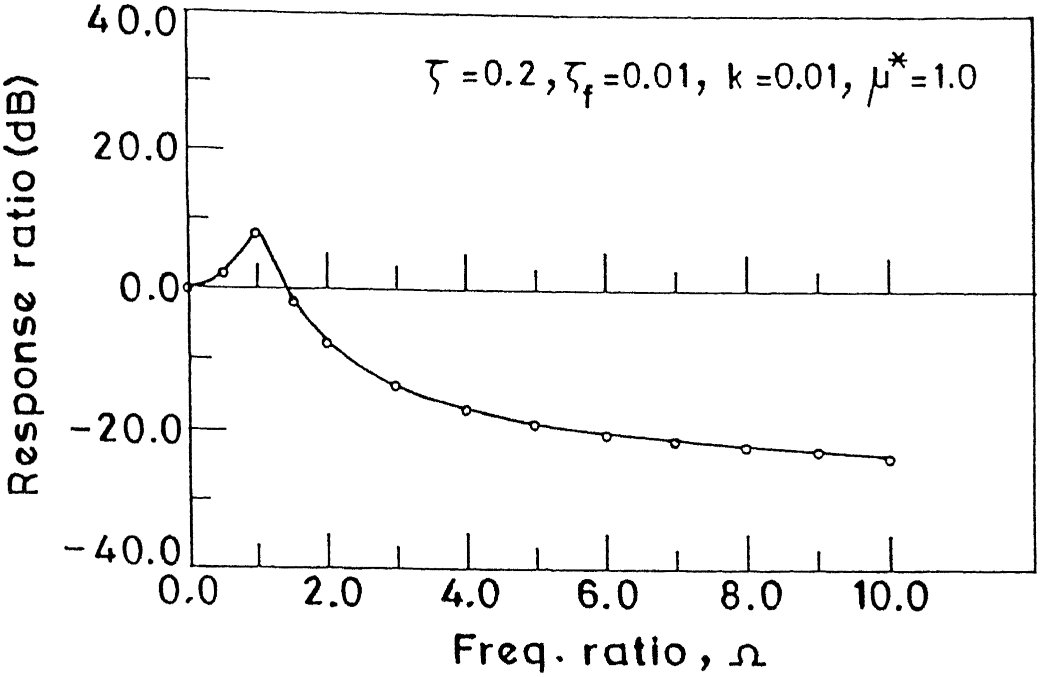


Fig.4.3(a) Response of mass-like foundation

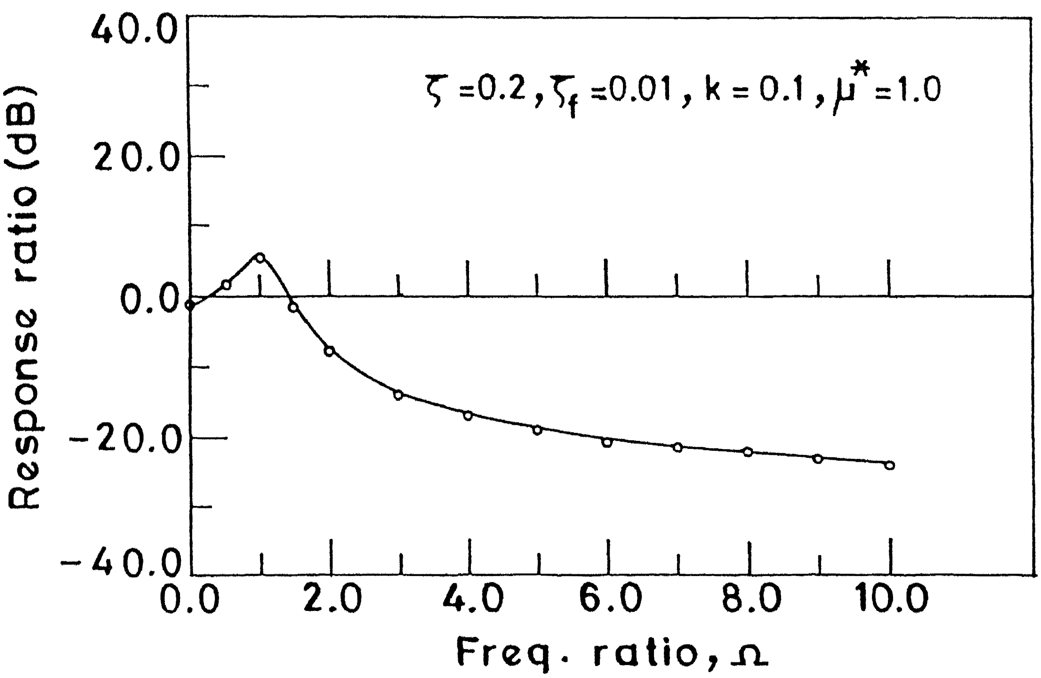


Fig.4.3(b) Response of mass-like foundation

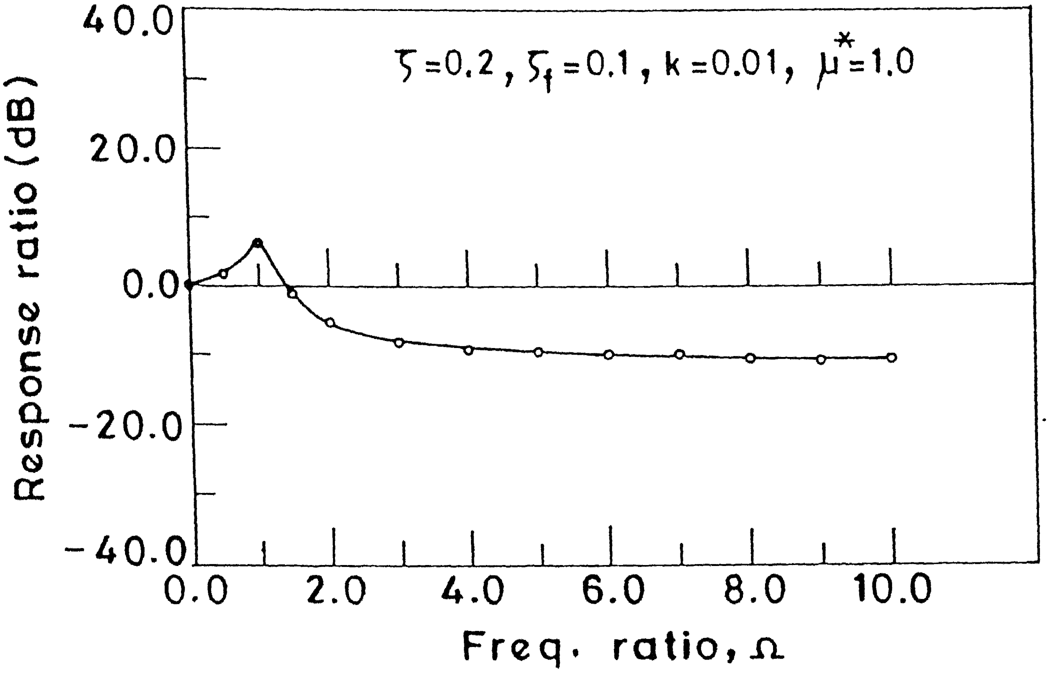


Fig.4.4(a) Response of mass-like foundation

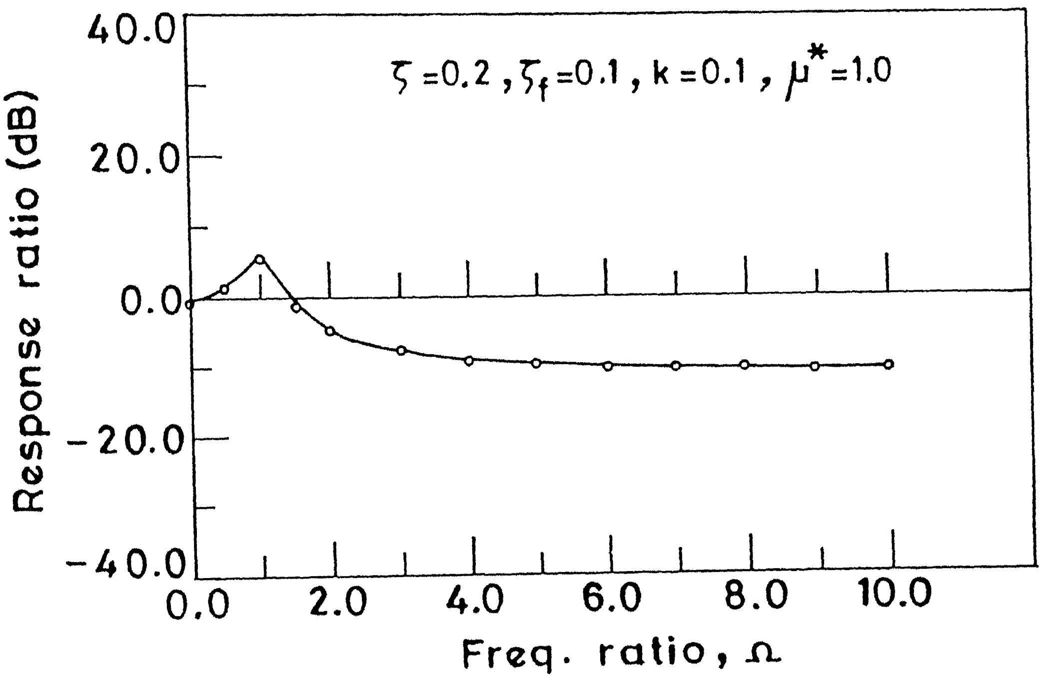


Fig.4.4(b) Response of mass-like foundation

other words, damping is harmful so far as isolation at high frequency is concerned.

The approximate method of harmonic balance used in the present analysis breaks down (as mentioned in chapter 3) when $\zeta > (\zeta_f)_c$ given by $(\zeta_f)_c = \frac{\mu^*}{2/\beta} = \frac{\pi}{8}\mu^*$. For an infinite mass foundation, $\mu^* = 1.0$ and so $(\zeta_f)_c = \frac{\pi}{8} = 0.39$. Den Hertog [17] presented the exact analysis for a linear restoring force and gave the expressions for finite resonant response when $\zeta > (\zeta_f)_c$. We can observe that with increase in the nonlinearity parameter k, the resonant response ratio decreases (Fig. 4.2).

4.2.2 Foundation with Finite Mass

If we take the foundation mass to be finite, then $\mu^* < 1$. Figures 4.5–4.8 show the response ratio vs. frequency ratio plots for finite mass foundations with $\mu^* = 0.8$. Results were obtained even for the case of $\mu^* = 0.5$ and 0.1 but are not presented here. Figures 4.5–4.6 show the results for $\zeta = 0$ and Figures 4.7–4.8 show the results for $\zeta = 0.2$.

On comparing Figures 4.5–4.8 respectively with Figures 4.1–4.4, we can point out the effect of foundation mass on response ratio. We find that $(\zeta_f)_c$ decreases with decreasing μ^* and so, the range of applicability of the harmonic balance gets further narrowed with respect to ζ_f for finite mass foundations. We can see that in general, with decrease in μ^*, the response ratio increases. In other words, the isolation of a finite impedance foundation is a harder task. The assumption of

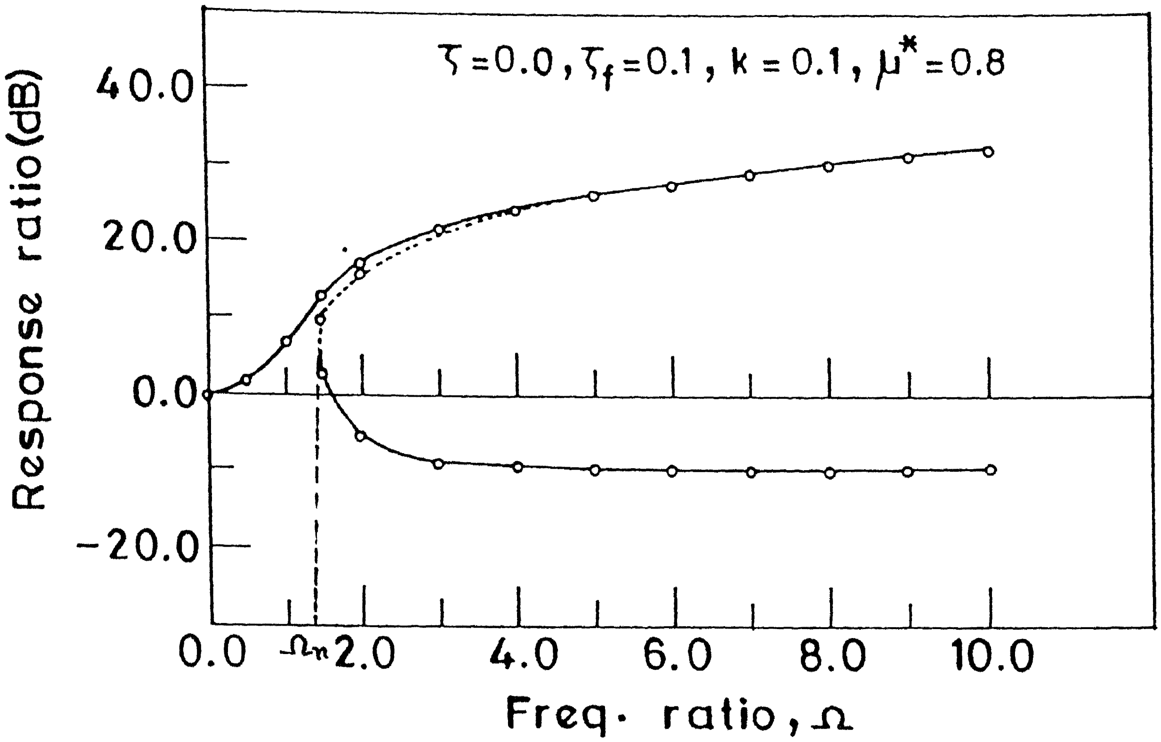


Fig.4.5(a) Response of mass-like foundation

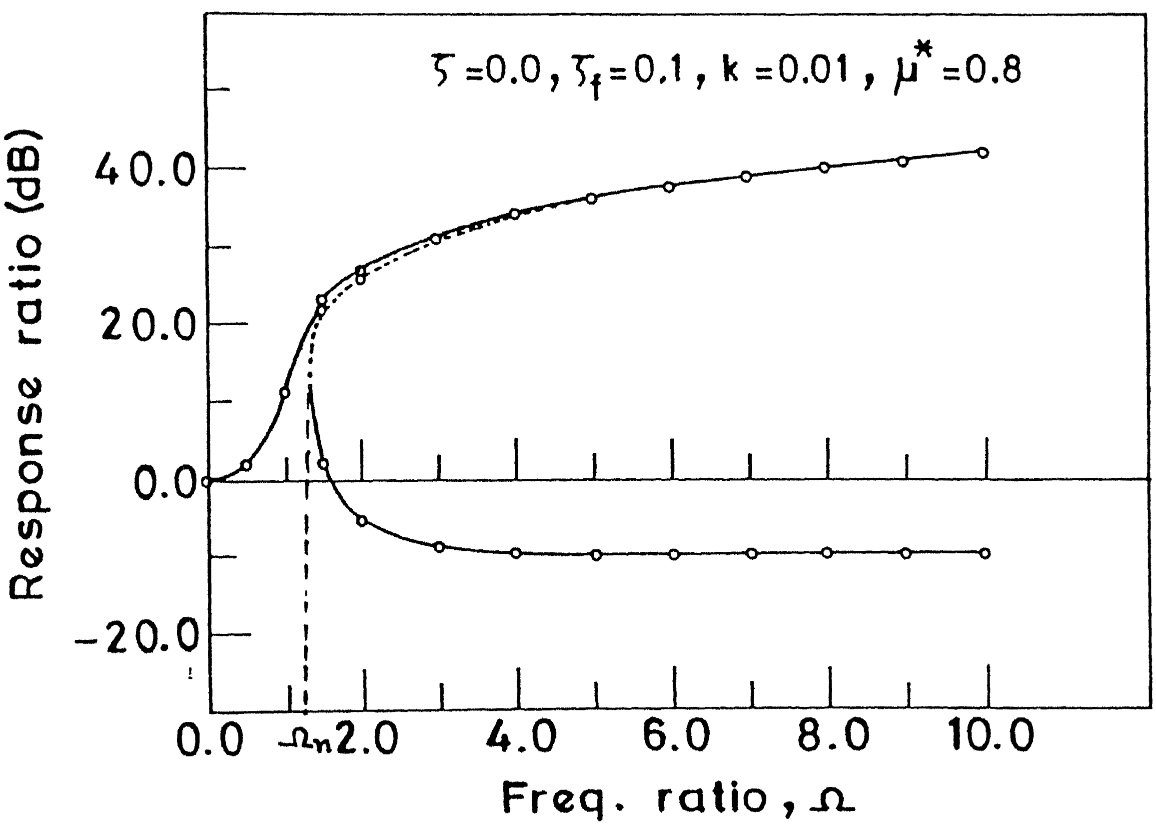


Fig.4.5(b) Response of mass-like foundation

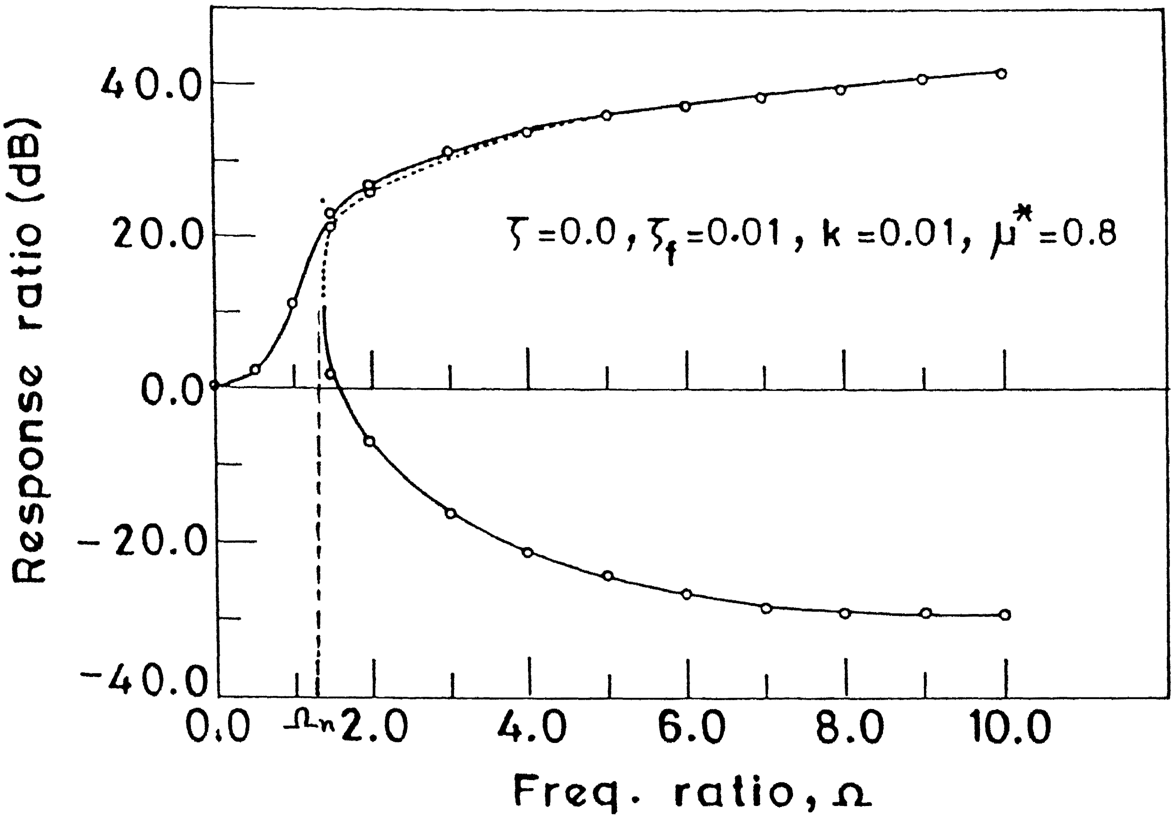


Fig.4.6(a) Response of mass-like foundation

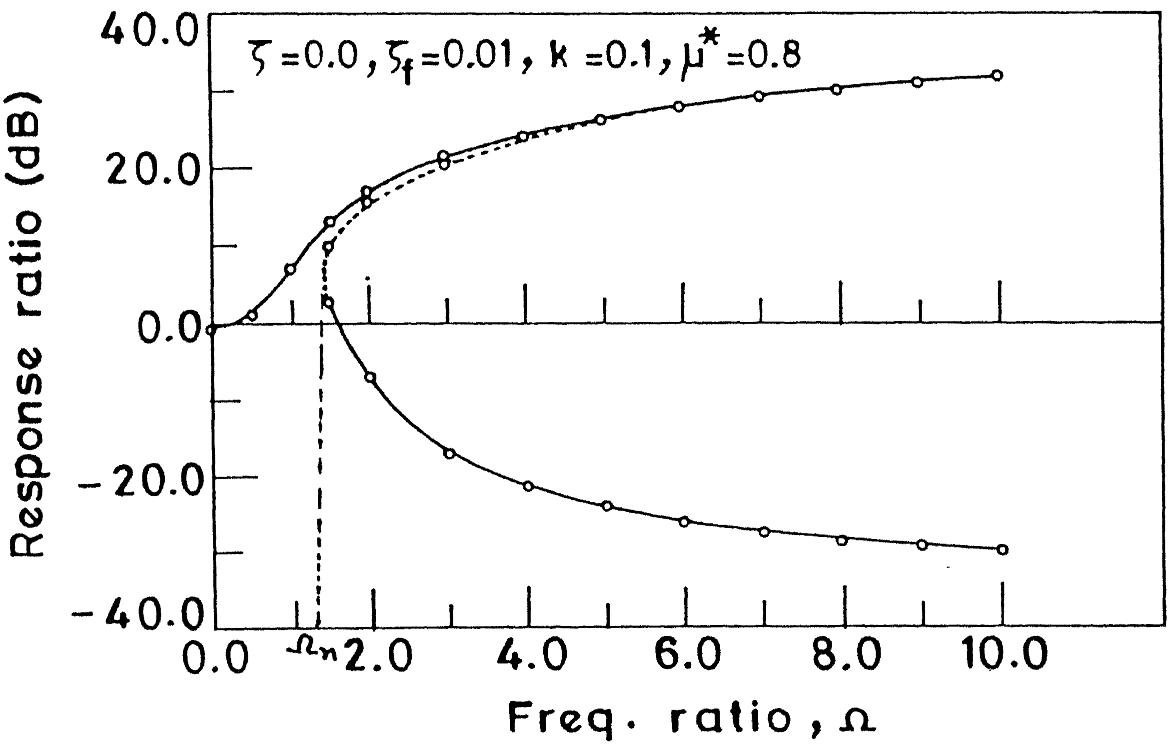


Fig.4.6(b) Response of mass like foundation

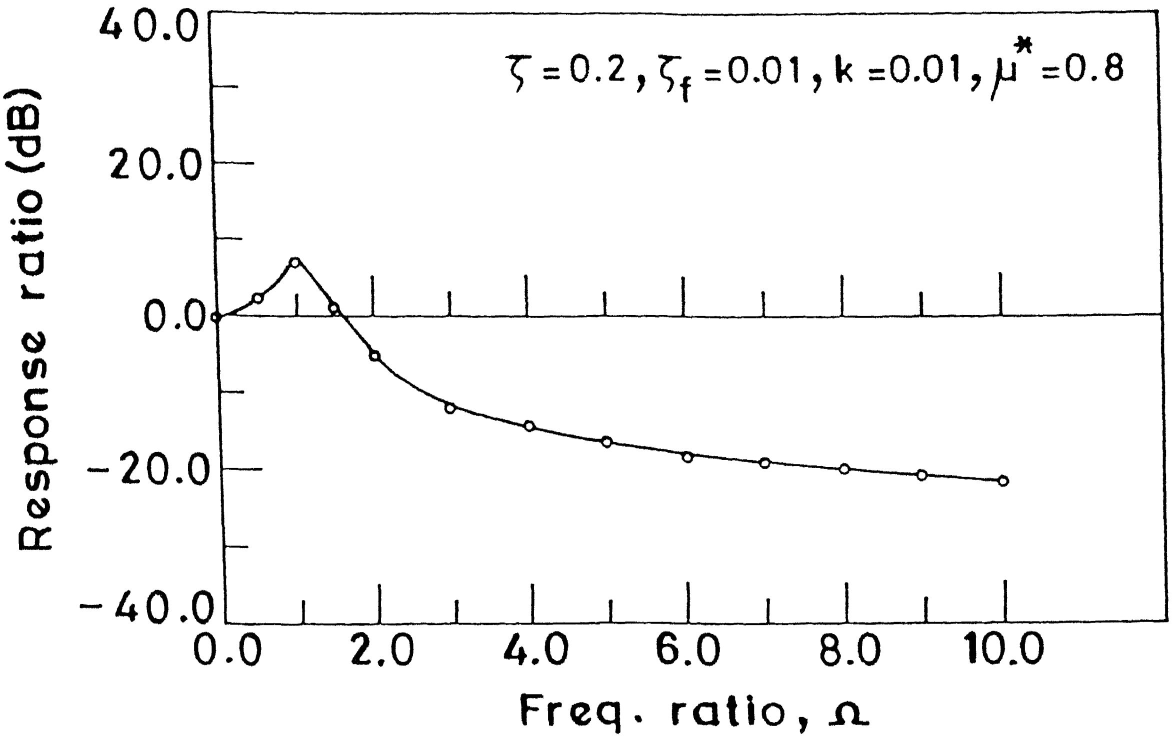


Fig.4.7(a) Response of mass-like foundation

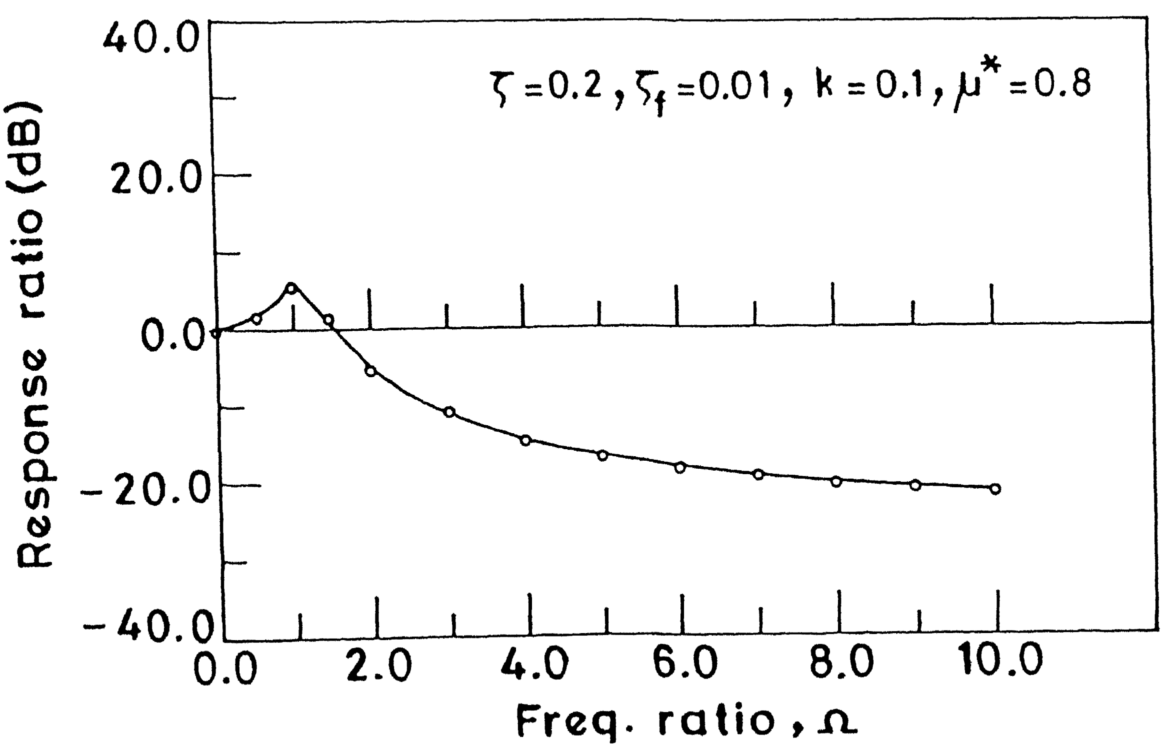


Fig.4.7(b) Response of mass-like foundation

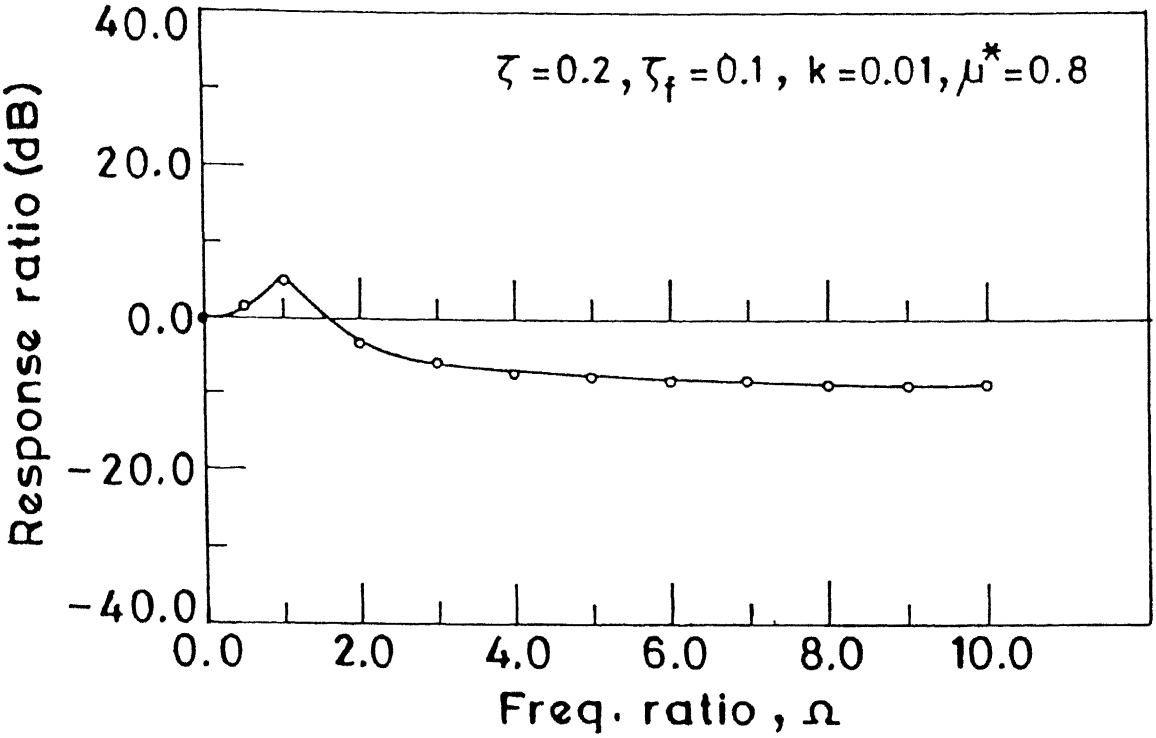


Fig.4.8(a) Response of mass like foundation

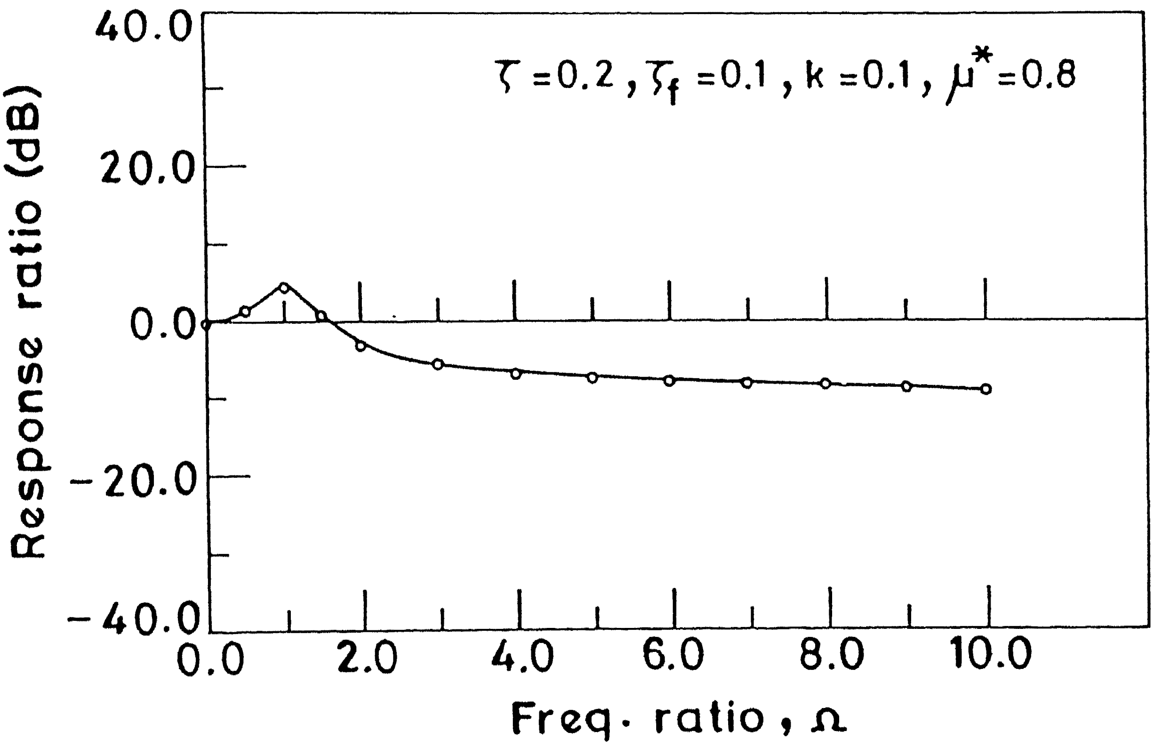


Fig.4.8(b) Response of mass-like foundation

infinitely rigid foundation over- estimates the performance of an isolator.

In the absence of viscous damping, the response ratio for μ^* = 0.8 is higher than for μ^* = 1.0 for fixed values of ζ_f and k, and the jump observed in Duffing's equation remains unchanged. In presence of viscous damping, the peak response (low frequency response) for μ^* = 0.8 is lower compared to μ^* = 1.0 (Figures 4.3-4.4 and Figures 4.7-4.8). But high frequency response for μ^* = 0.8 is higher compared to μ^* = 1.0. An increase in ζ_f always increases the high frequency response in both the cases (for $\zeta \neq 0$) but the response still remains higher in case of μ^* = 0.8, as compared to μ^* = 1.0.

It was verified by computation that the type of deviation in the result for μ^* = 0.8 with respect to μ^* = 1.0 remains unaltered for μ^* = 0.5 with respect to μ^* = 0.8.

The turning point frequency Ω_n (Fig. 4.1(a)) in the absence of viscous damping is obtained from eqn. (3.1-30). The values so obtained are in perfect agreement with those indicated in Figures 4.1, 4.2, 4.5 and 4.6.

4.3 PLATELIKE FOUNDATIONS

Figs. 4.9-4.14 show the plots of response ratio (in dB) vs. frequency ratio for platelike foundations. The parameter μ^* resulting in masslike foundation case has been replaced by the impedance ratio parameter p. In order to compare the results of platelike foundations with masslike foundations, the information outlined in references [5-8] was used to find the value of p for

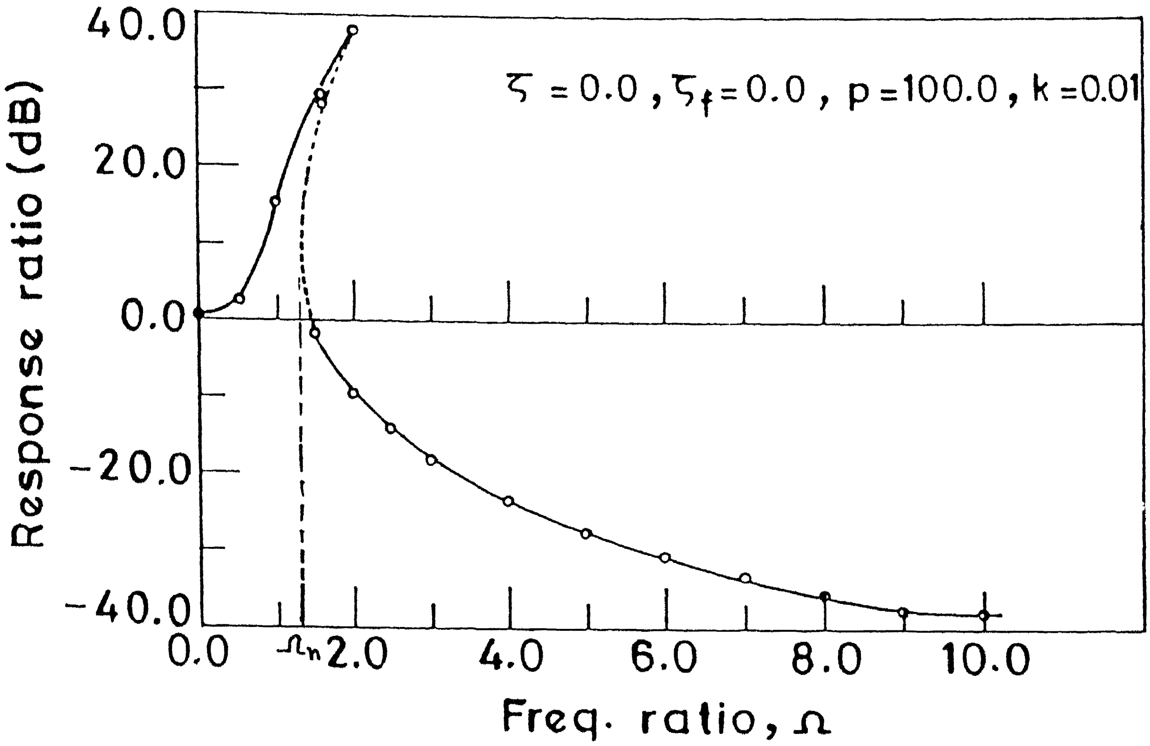


Fig.4.11(a) Response of plate-like foundation

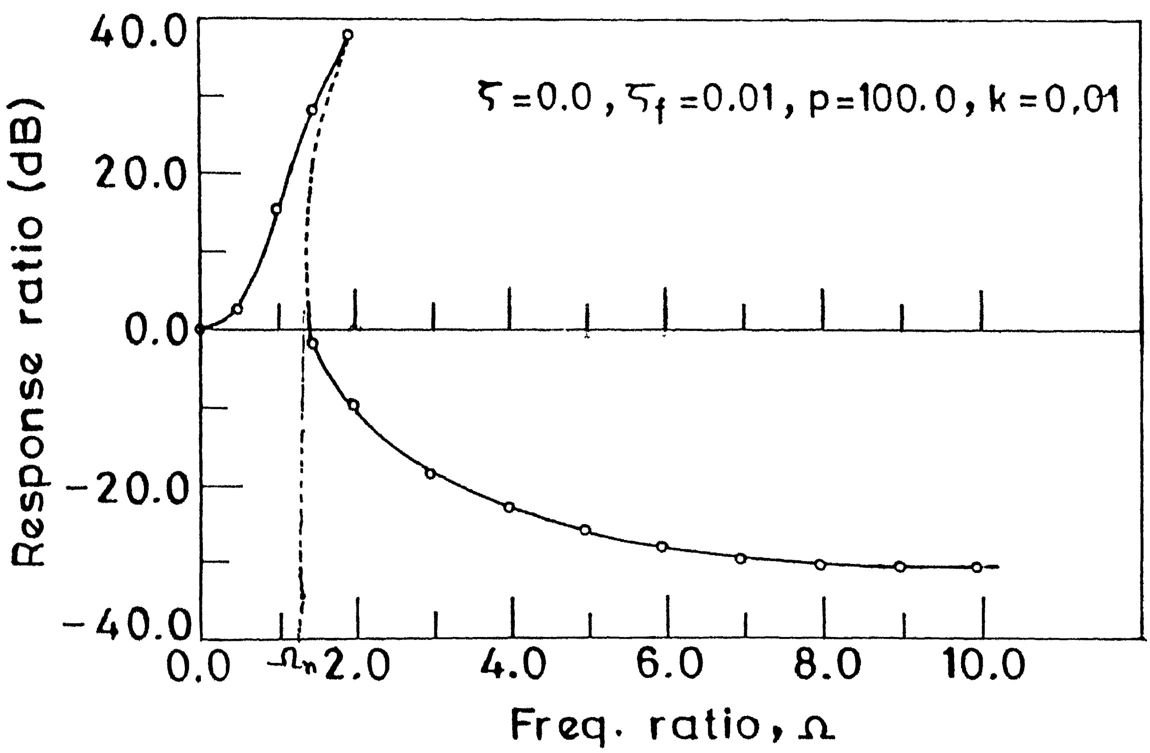


Fig.4.11(b) Response of plate-like foundation

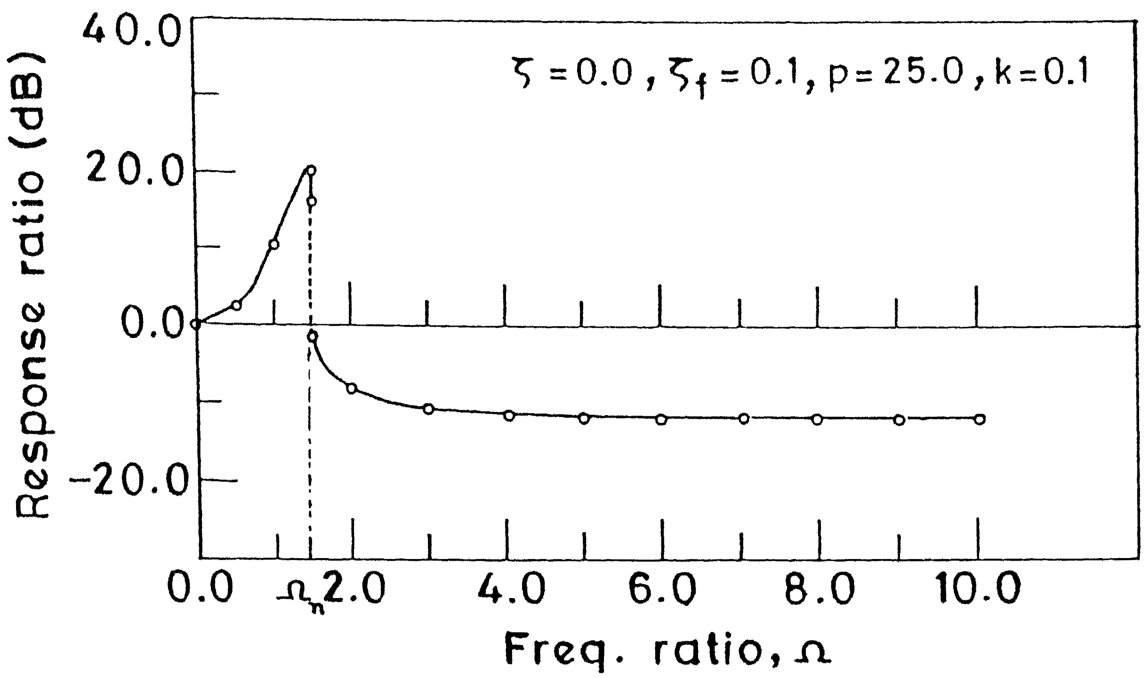


Fig. 4.10(a) Response of plate-like foundation

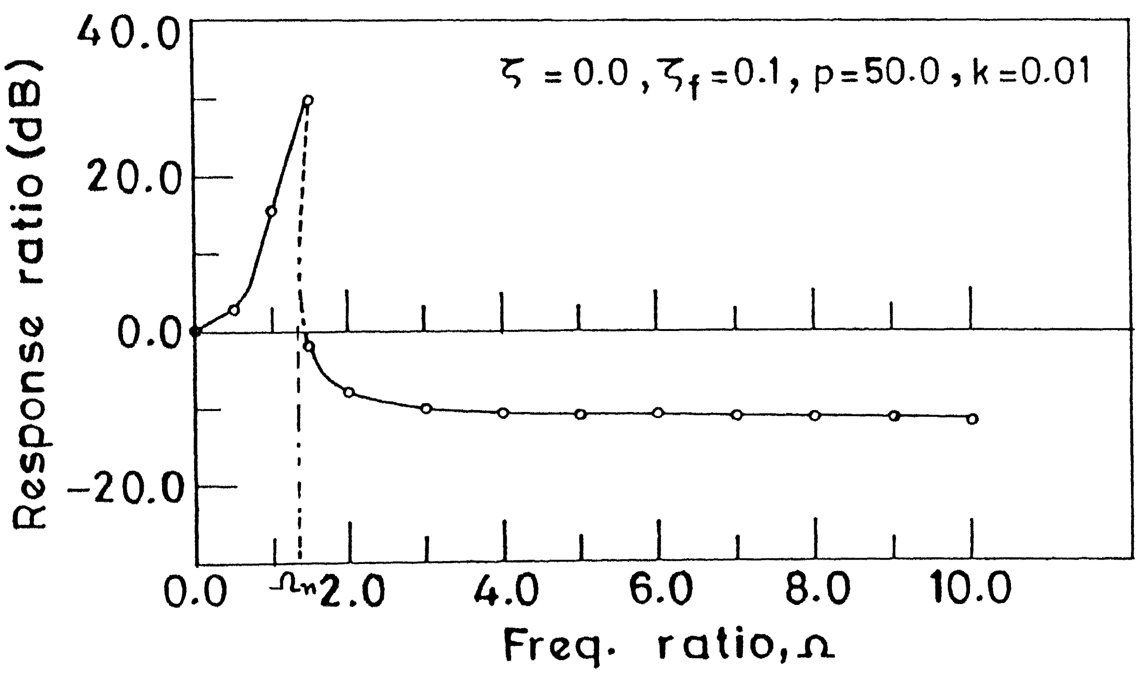


Fig. 4.10(b) Response of plate-like foundation

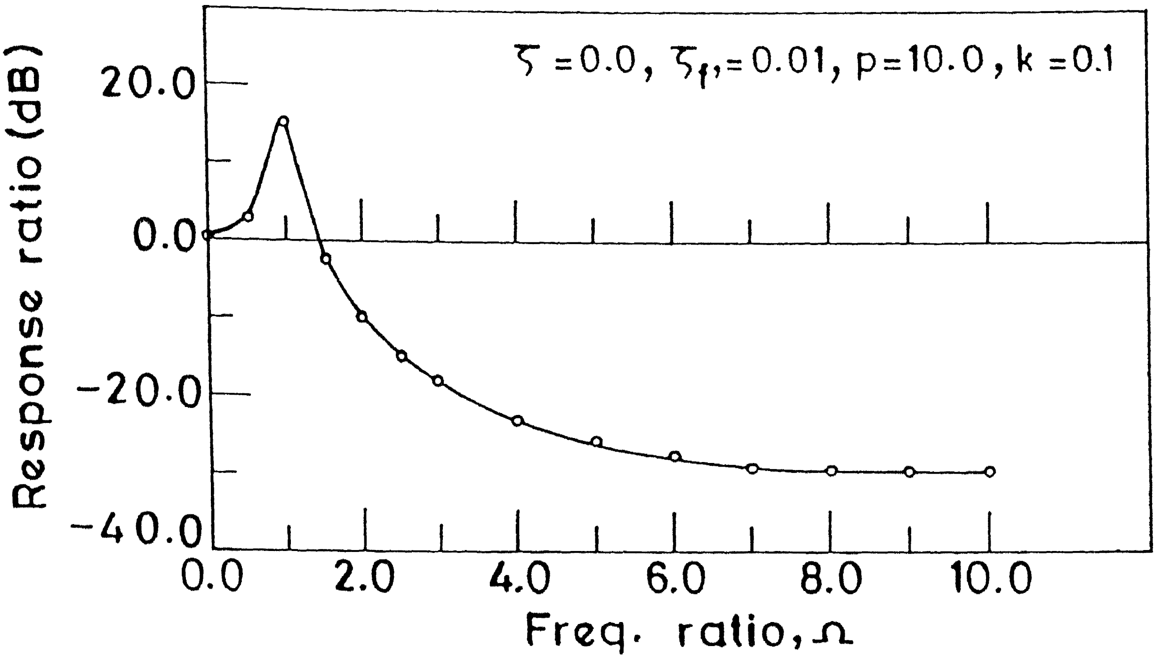


Fig.4.9(a) Response of plate-like foundation

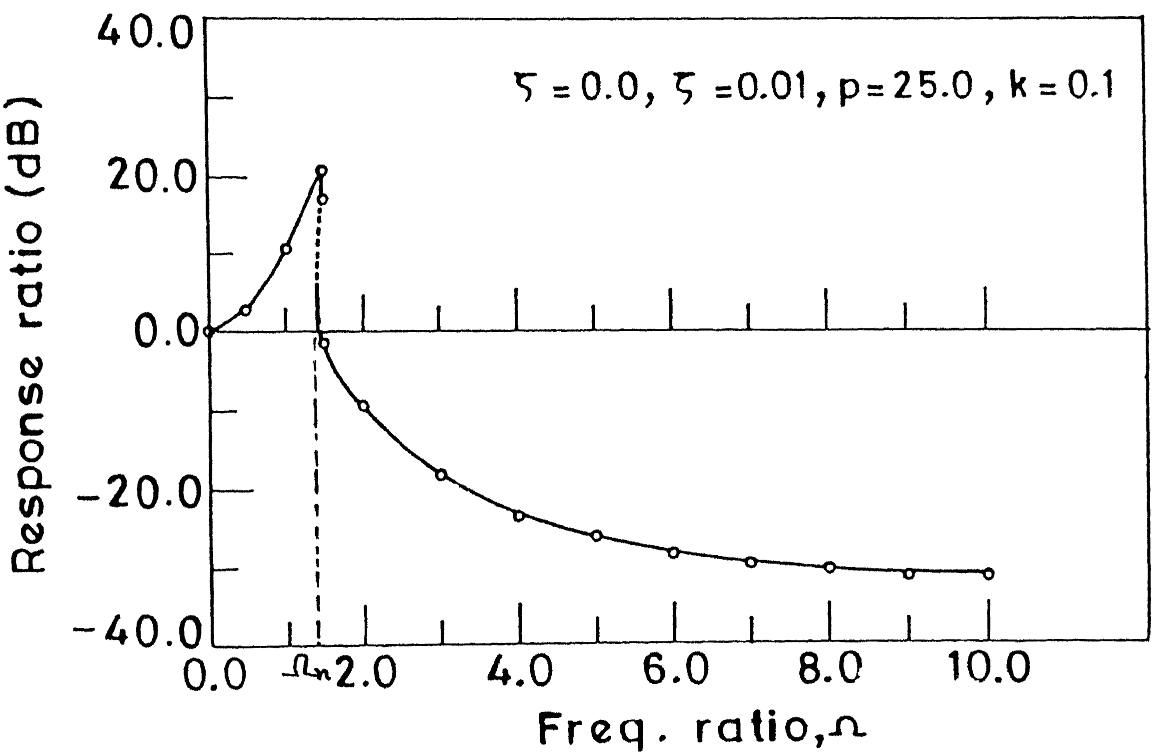


Fig.4.9(b) Response of plate-like foundation

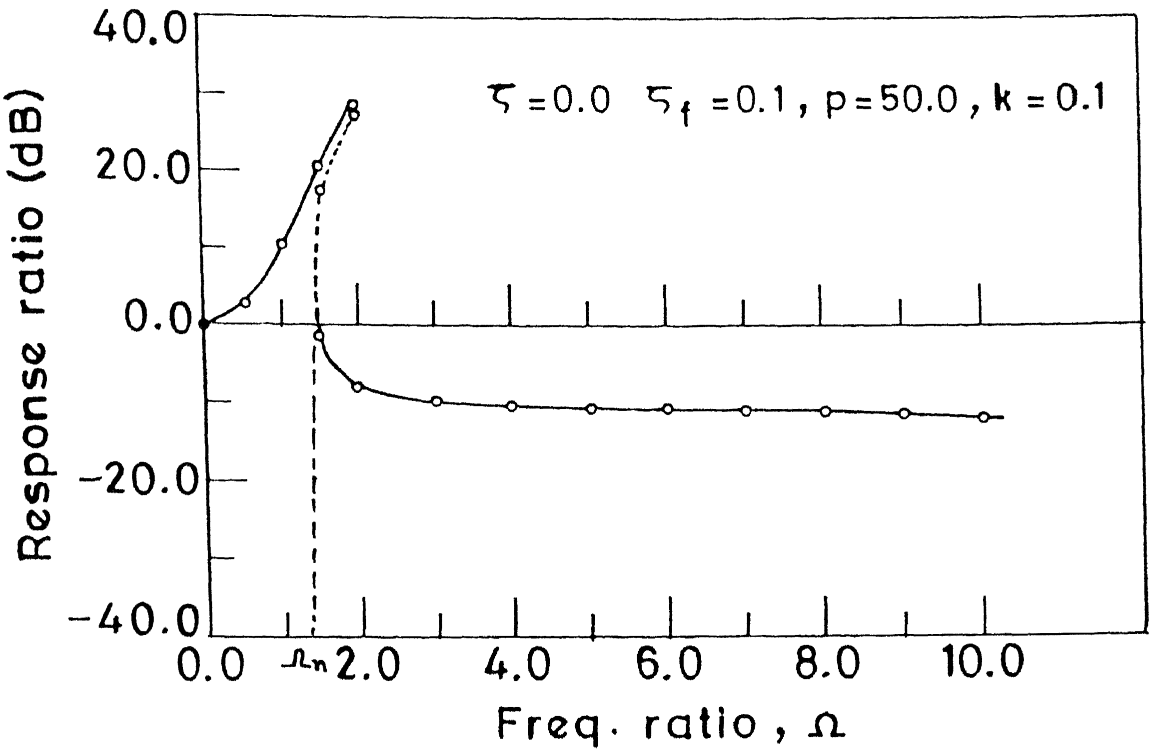


Fig.4.12(a) Response of plate-like foundation

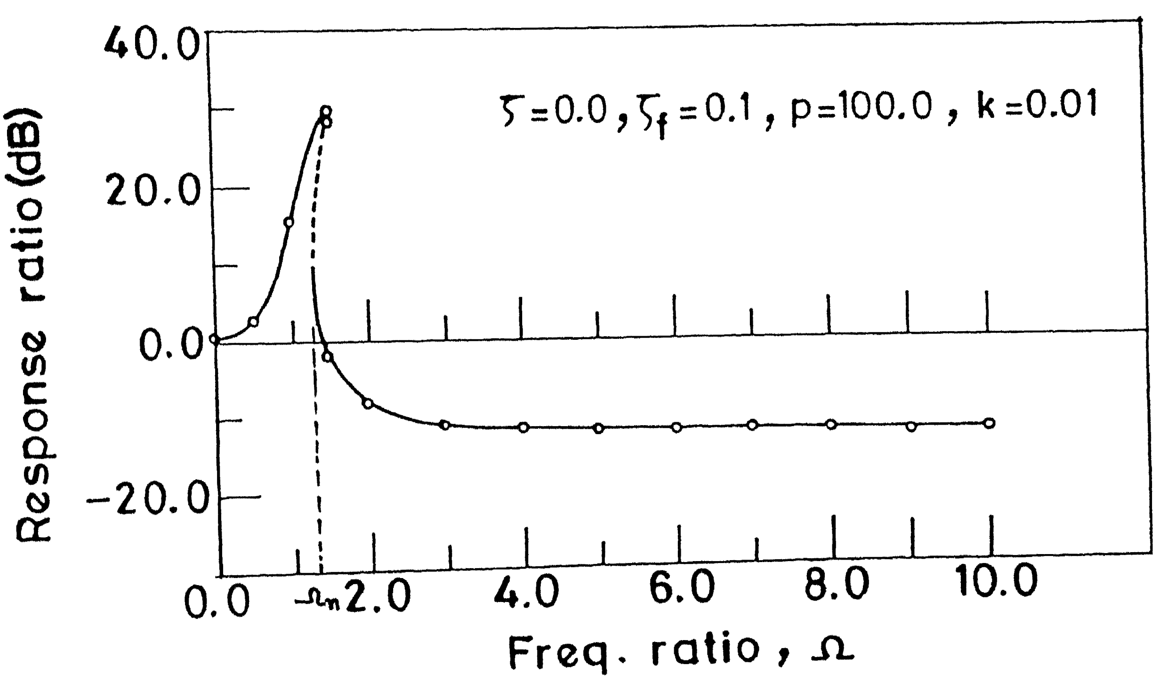


Fig. 4.12(b) Response of plate-like foundation

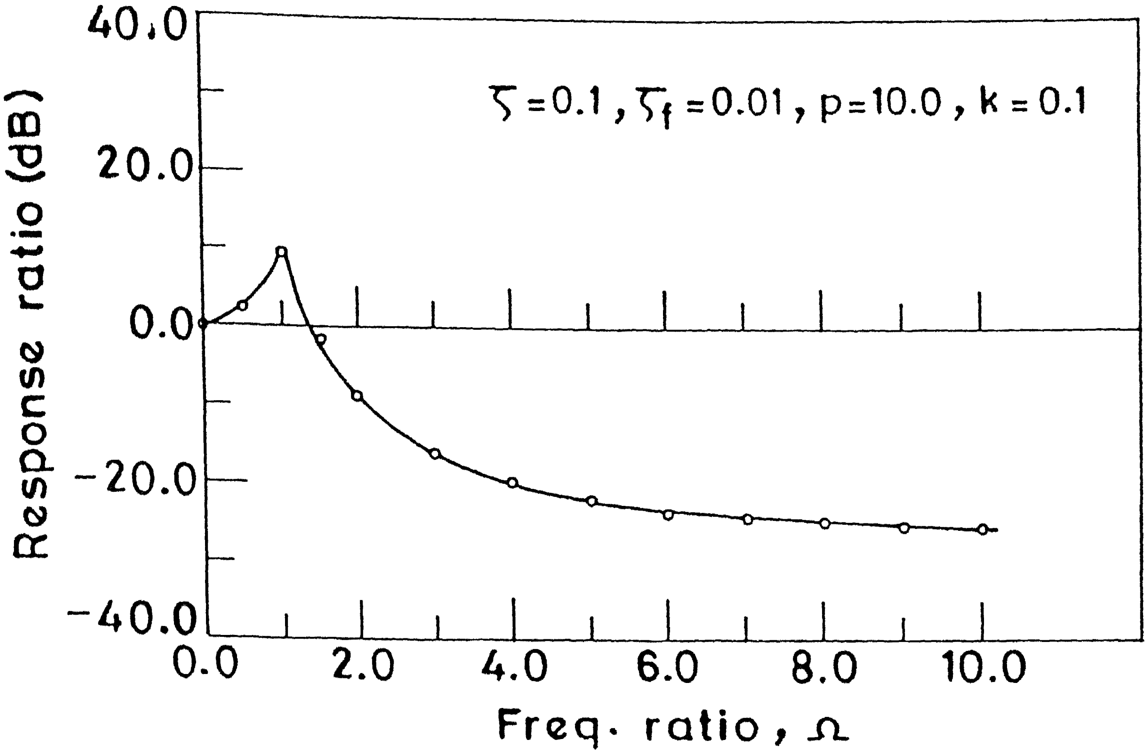


Fig.4.13(a) Response of plate-like foundation

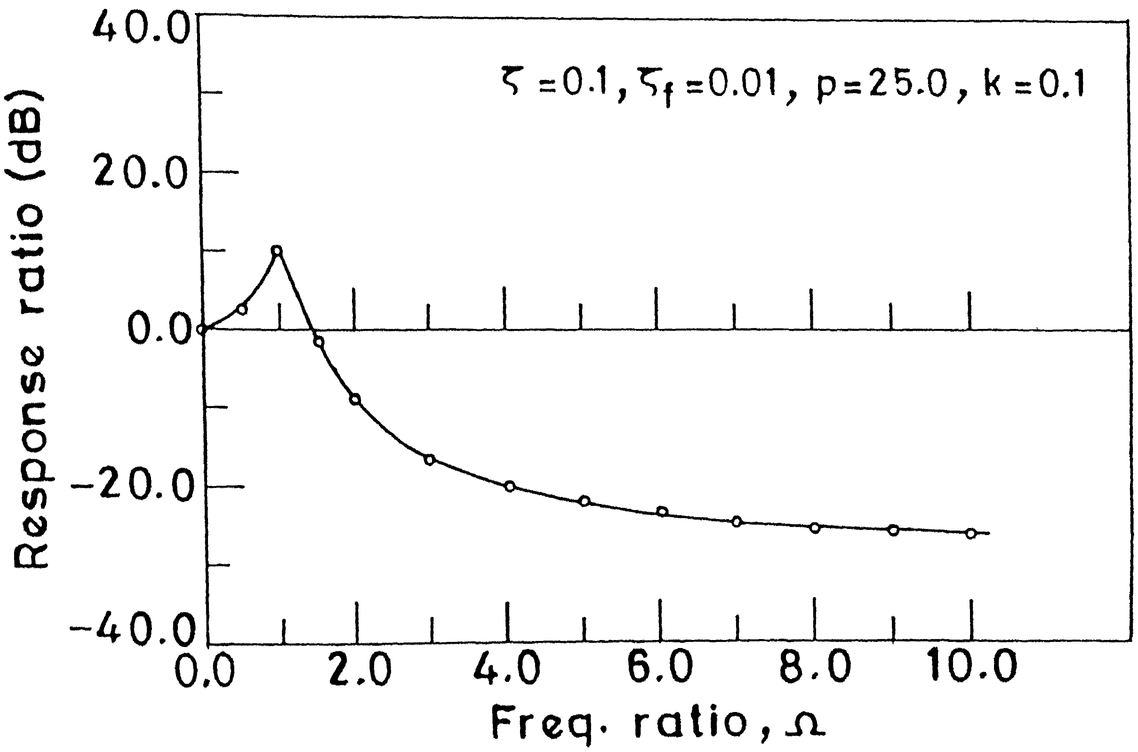


Fig.4.13(b) Response of plate-like foundation

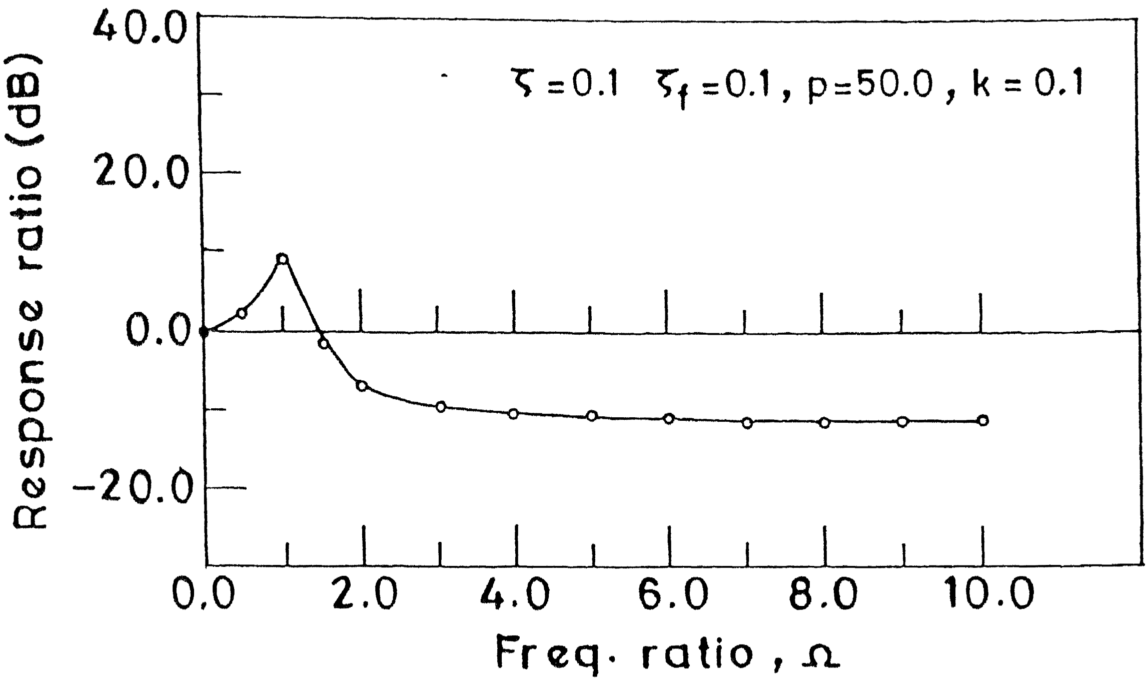


Fig.4.14(a) Response of plate-like foundation

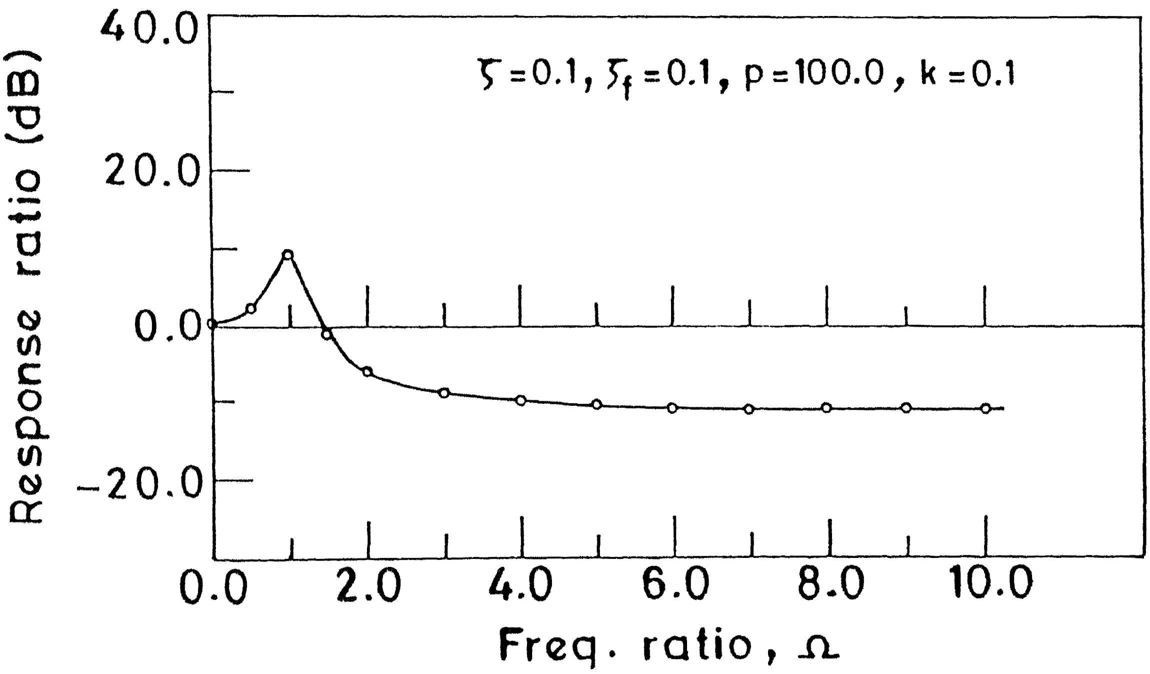


Fig.4.14(b) Response of plate-like foundation

plates of practical dimensions and mass. If the plate material is taken to be either aluminium or steel, it can be found that the reasonable values of p range from 10 to 50. If $p \rightarrow \infty$, we will get the results of a plate having infinite impedance.

The approximate method of harmonic balance is not valid for the case when $p < p_c$ where p_c is the value of p at which the constant term in eqn. (3.2-15) vanishes for a given value of Ω and ζ_f. Actually, from eqn. (3.2-15), it can be seen that for that value of the parameter p (when other parameters are unaltered) for which the term independent of B_0 becomes greater than zero, there is no response and the method of harmonic balance does not remain valid.

Figures (4.9-4.12) show the response curves in absence of viscous damping. In this case, beyond certain value of Ω (Ω_n), there are multiple solutions existing. Although we do not have analytical expressions for a verification, even then with the information that the values of Ω_n obtained analytically as well as numerically for a masslike foundation matched, we can expect that those obtained for a platelike foundation are also correct. It can be seen that for $\zeta = 0$, for a given value of ζ_f and k, an increase in p shows the existence of multiple roots (the jump observed in Duffing's oscillator). For larger values of p, the post jump region has three branches. It can be seen that the nonlinearity parameter k has very little effect on the nature of the response but an increase in k widens the jump region. The high frequency response remains almost unaltered with increase in k.

Figures (4.13-4.14) show the response curves in presence of viscous damping (ζ = 0.1) and it is obvious that the jump vanishes for such a high value of ζ. With an increase in the value of p, we find that the peak response increase but the high frequency response decreases. An increase in the nonlinearity parameter decreases the peak response ratio but does not affect the high frequency response ratio.

On comparison, we can see that the high frequency response of a platelike foundation is lower than that of pure masslike foundation, whereas low frequency peak responses for both are almost of the same order and decrease with increasing k.

4.4 BEAMLIKE FOUNDATIONS

Figures 4.15–4.20 show the response of beamlike foundations. Figures 4.15-4.17 are the response ratio plots in absence of viscous damping whereas Figures 4.18–4.20 are in presence of viscous damping. The existence of multiple solution or turning point can be observed in the graphs with ζ = 0. Since analytical expressions are not available for verification, as argued for platelike foundation we will accept the values of nonresonant critical frequency (Ω_n) appearing in the response curve (Figures 4.15-4.17) to be correct. We can compare these responses with those of platelike foundations. Even though the values of impedance ratio (p) are taken to be same for both platelike and beamlike foundations, due to difference in the method of nondimensionalizing their response equations, they cannot be taken to be numerically equivalent. With the help of information in

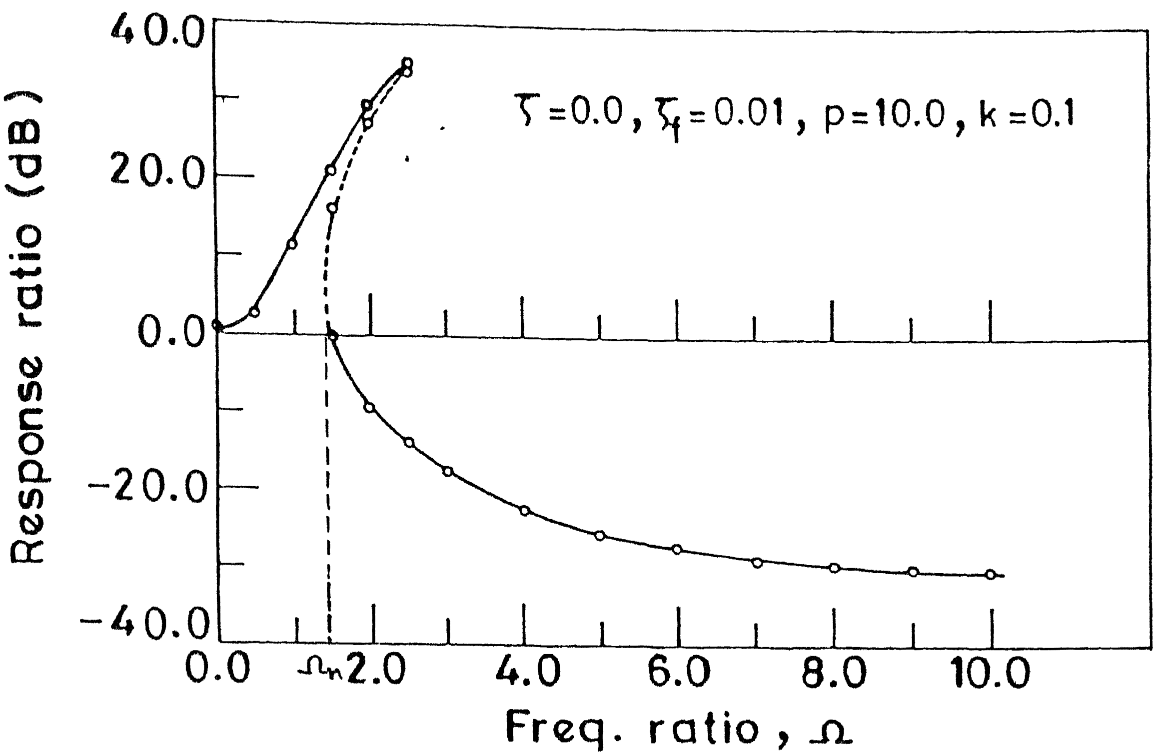


Fig.4.15(a) Response of beam-like foundation

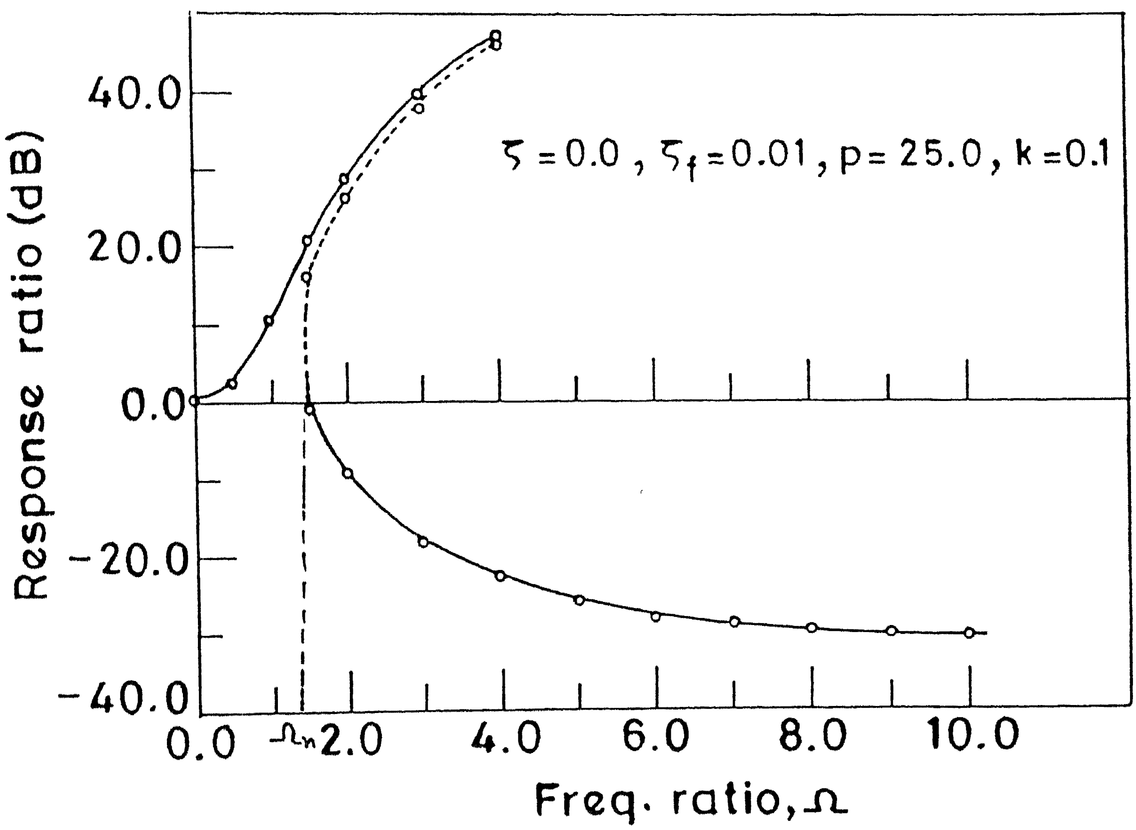


Fig.4.15(b) Response of beam-like foundation

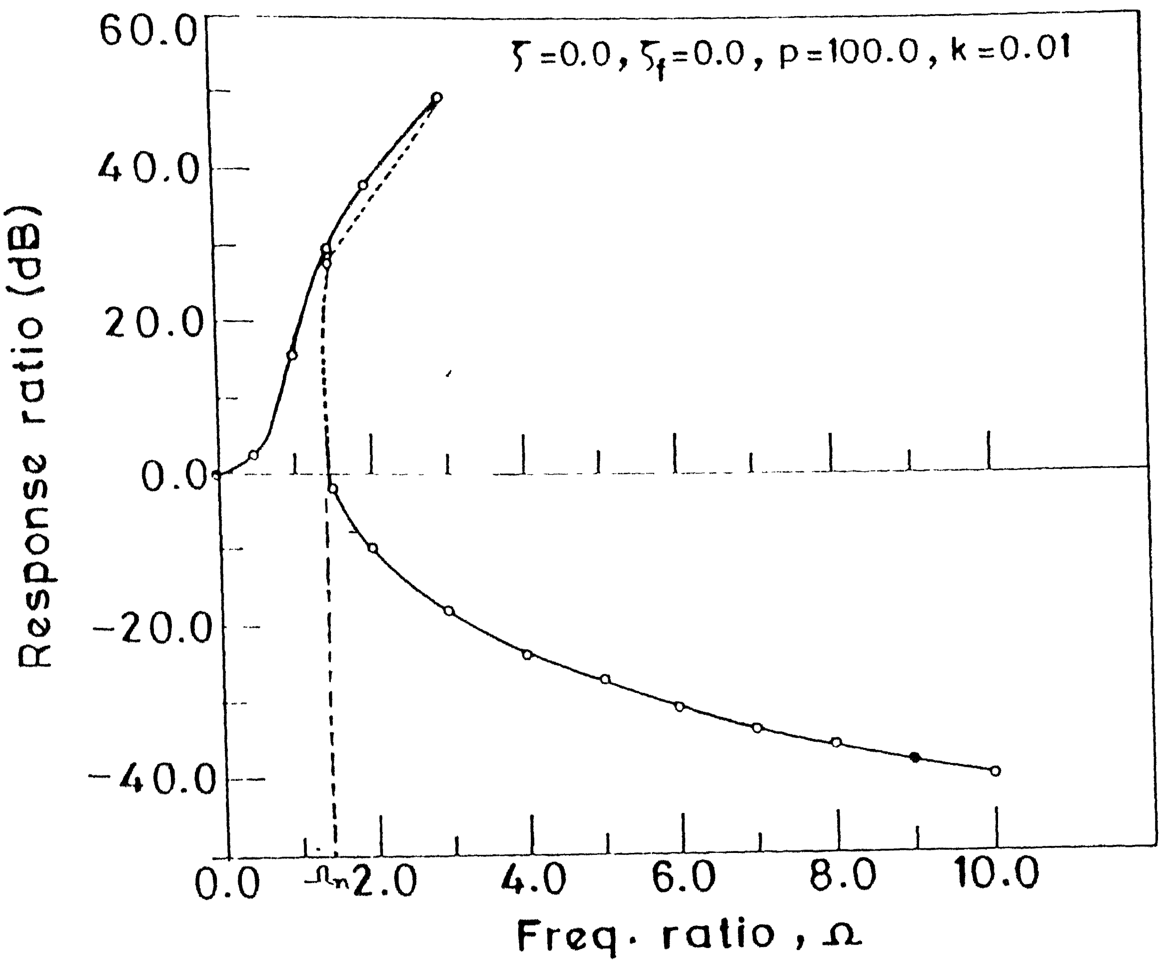


Fig.4.17(a) Response of beam-like foundation

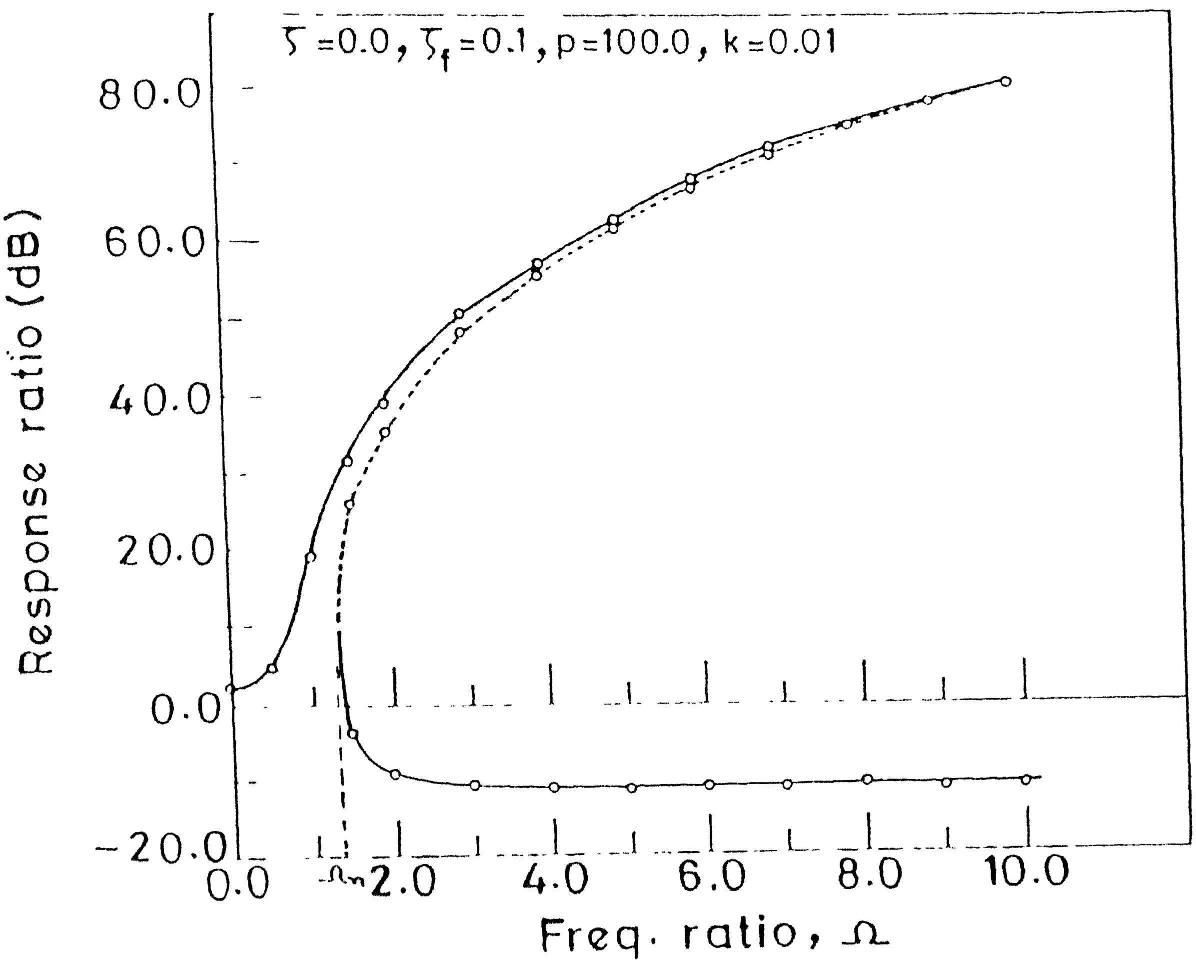


Fig.4.17(b) Response of beam-like foundation

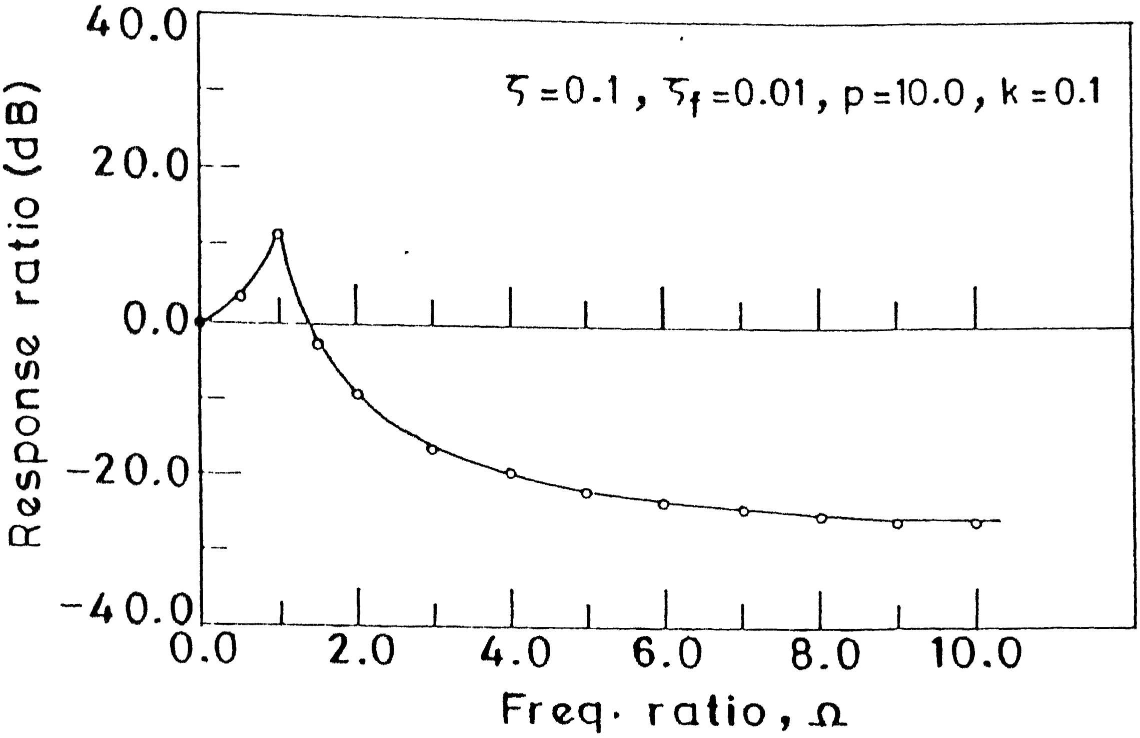


Fig.4.18(a) Response of beam-like foundation

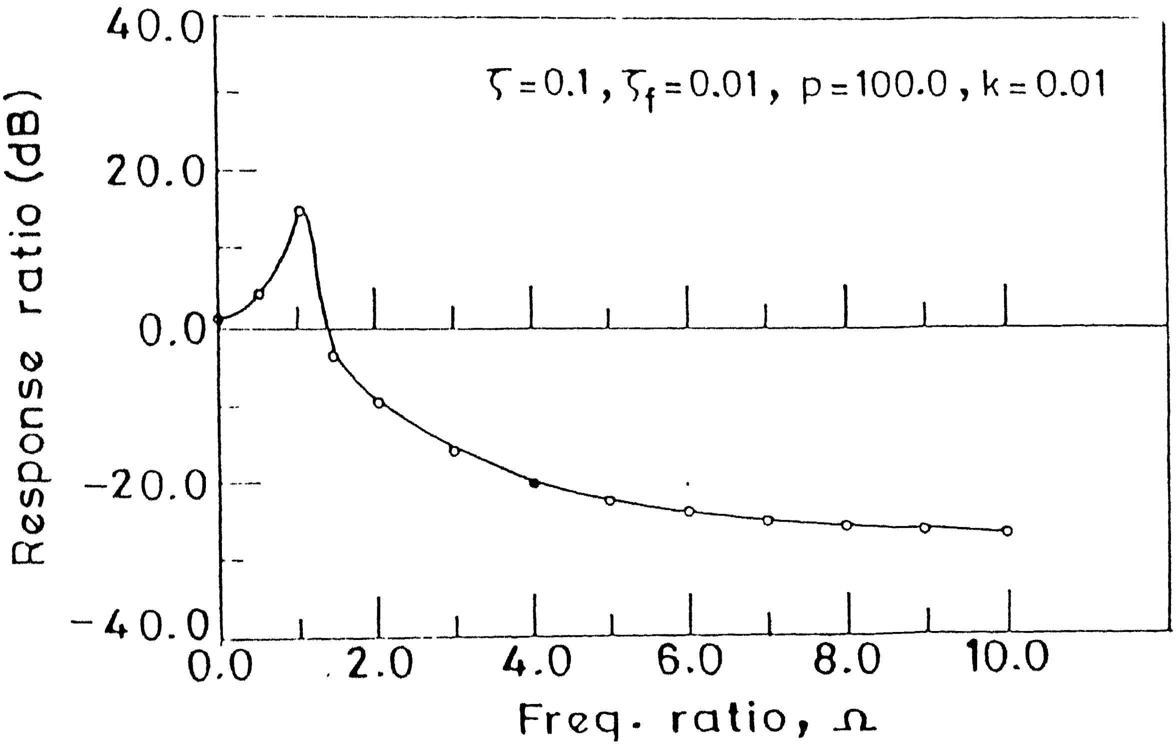


Fig.4.18(b) Response of beam-like foundatic

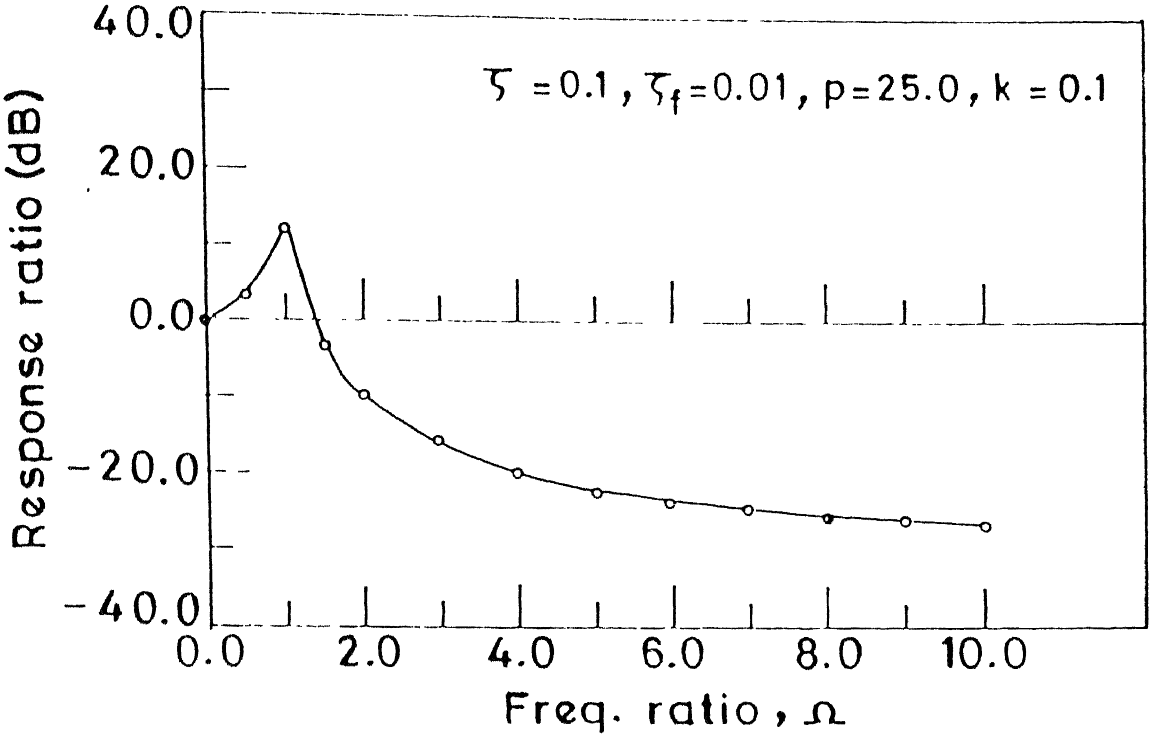


Fig.4.19(a) Response of beam-like foundation

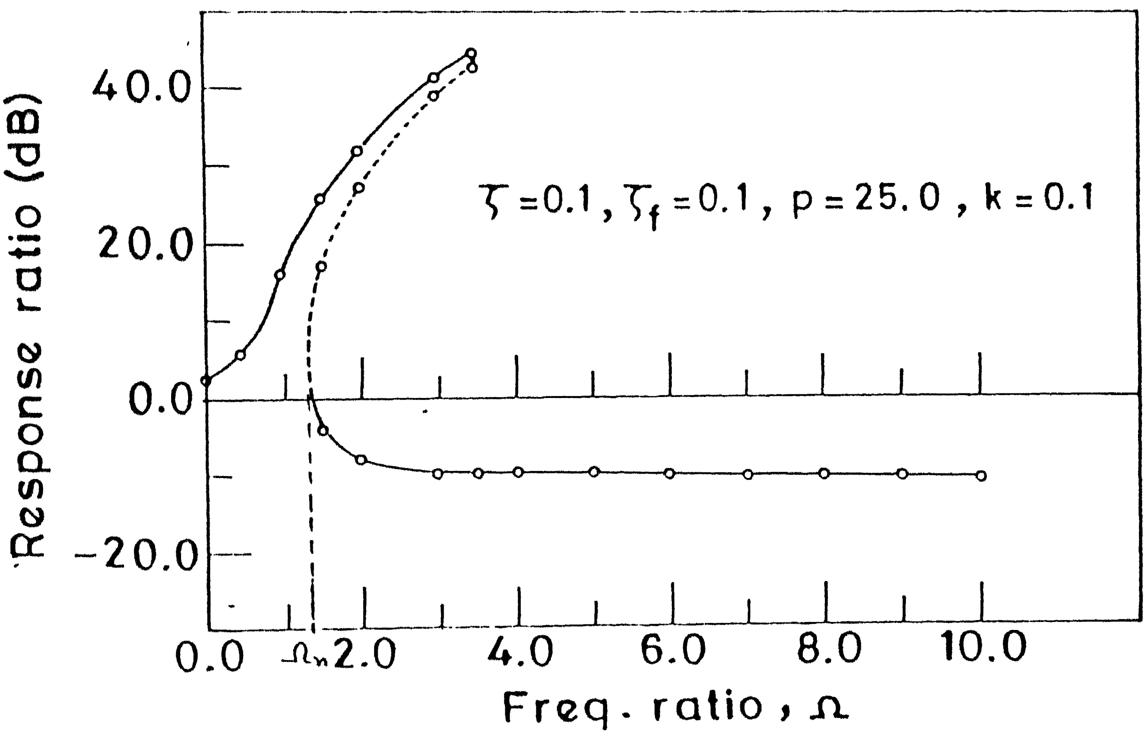


Fig.4.19(b) Response of beam-like foundation

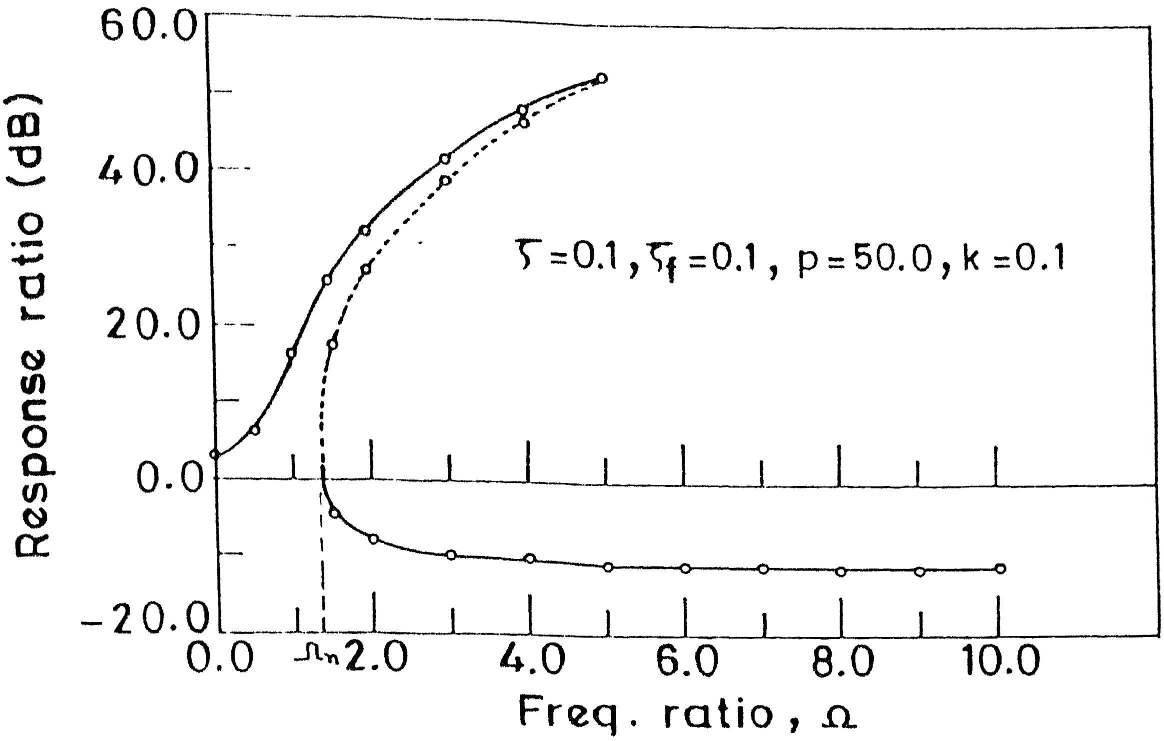


Fig.4.20(a) Response of beam-like foundation

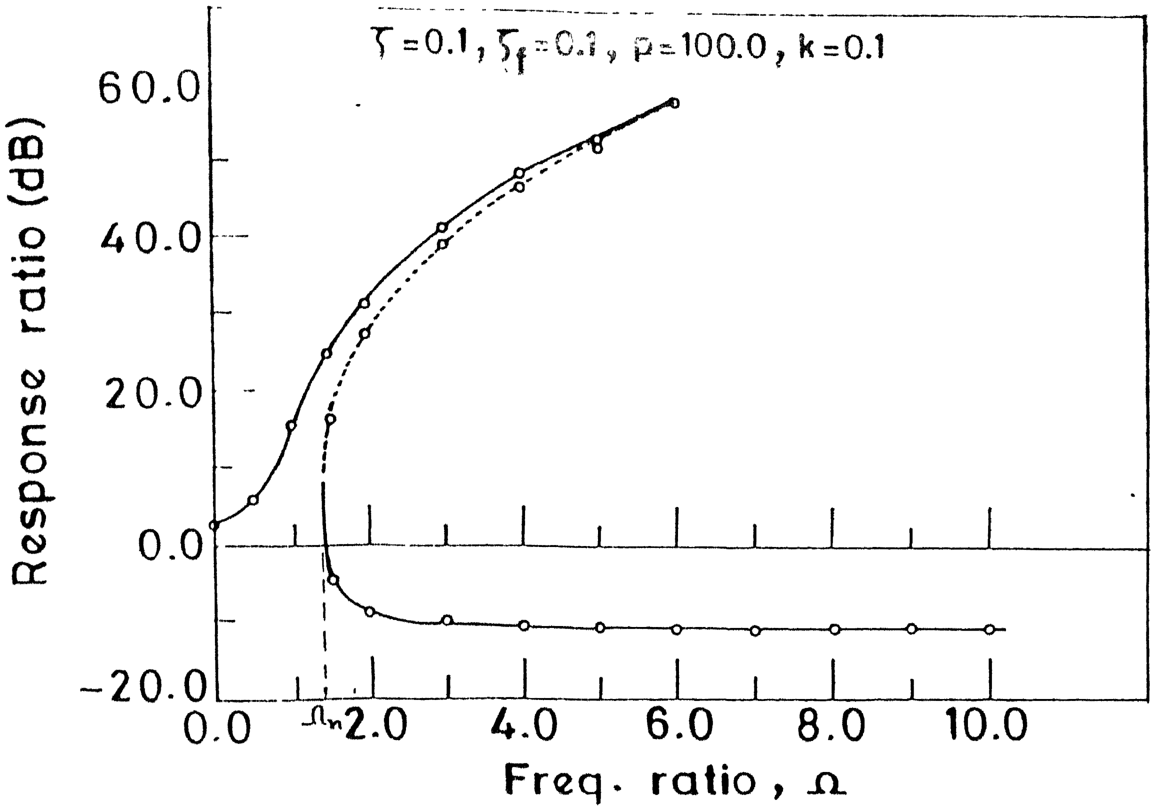


Fig.4.20(b) Response of beam-like foundation

reference [6-8], for a beam of usual dimensions, the values of p range from 5 to 50 (for aluminium as well as steel beams). To reduce the computational labour, in both the cases of platelike and beamlike foundations, the response ratio are studied for same set of values of other parameters.

Figures 4.15-4.17 show that in absence of viscous damping, the bifurcation and multiple values of response ratio in high frequency region are more pronounced here when compared to platelike foundations. The resonant response is steeper compared to both masslike and platelike foundations, but the high frequency stable response remains almost same. From Figures 4.18-4.20, we can see that the jump persists even in presence of viscous damping with $\zeta = 0.1$. It can be seen that for $\zeta = 0.1$, $\zeta_f = 0.01$, $p = 10.0$, $k = 0.1$ (Fig. 4.18a) there is no jump but with an increase in p and/or k, the jump appears.

Just as discussed for platelike foundations, there exists a critical value of parameters ζ_f as well as p for every Ω when the constant term in eqn. (3.3-9) becomes greater than zero. For $\zeta_f > (\zeta_f)_c$ and $p < p_c$, the method of harmonic balance does not remain valid. The expression of $(\zeta_f)_c$ implicit with p and Ω was given in eqn. (3.3-10).

CHAPTER V

CONCLUSIONS AND SCOPE OF FURTHER WORK

5.1 CONCLUSIONS

This thesis extends the work of Ravindra [13] and Den Hertog [17] on isolators with combined viscous and Coulomb damping by including a linear restoring force over the nonlinear term with the foundation having been modelled as masslike, platelike and beamlike and the performance characteristics of such isolators are presented through a detailed parametric study. Addition of viscous damping controls the jump and thereby enhances the range of frequency over which the response ratio can be maintained at less than unity. The analytical results for all the cases have been derived and the effect of decreasing the foundation impedance on response ratio is shown by computing the nonlinear algebraic equations numerically. The analysis is done by assuming continuous motion and hence it is valid strictly for low damping cases.

It is found that with decreasing impedance of the foundation, the effectiveness of the isolator is better at low frequency values whereas at high frequency values it tends to become poorer. An increase in the nonlinearity parameter in the isolator

decreases the resonant response of the isolator for both finite as well as infinite impedance foundations. Also, it is shown that a simultaneous increase in Coulomb damping as well as viscous damping eliminates the resonant response (the jump behaviour usually observed in hard Duffing oscillator). For masslike foundations, expression derived analytically for nonresonant critical frequency (Ω_n) (beyond which multiple solutions for response ratio exist), are found to match with those obtained from response curves by numerical computation. For platelike and beamlike foundations, it is difficult to derive the expression for Ω_n and so those obtained from response curves are taken to be correct. For platelike and beamlike foundations it is seen that an increase in the impedance of the foundation enhances the jump width even in presence of viscous damping. So foundations with finite impedance are superior to one with infinite impedance if jump is to be avoided.

5.2 SCOPE OF FURTHER WORK

It is well known that the secondary resonances and chaos occur on the low-frequency regime around the jump in the resonant response curve of a hard Duffing equation with linear damping. Hence, it will be interesting to find out the effects of friction damping on these phenomena if these can be eliminated due to presence of the same. Also, the interaction of the elasticity and the distributed mass of the foundation at high frequencies could be of due consideration. Hence, the finite foundation dimension and boundary resonances deserve a special attention as extension of present work.

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